MASTER :)

13-3 Trigonometric Functions of General Angles

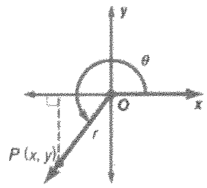
Trigonometric Functions for General Angles: You can find values of trigonometric functions for any angle that is greater than 90° or less than 0°.

KeyConcept Trigonometric Functions of General Angles

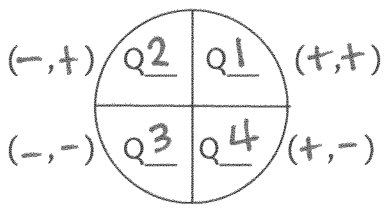
Let θ be an angle in standard position and let $P(x, y)$ be a point on its terminal side. Using the Pythagorean Theorem, $r = \sqrt{x^2 + y^2}$. The six trigonometric functions of θ are defined below.

$\sin\theta = \frac{y}{r}$ $\quad\cos\theta = \frac{x}{r}$ $\quad\tan\theta = \frac{y}{x}, x \neq 0$

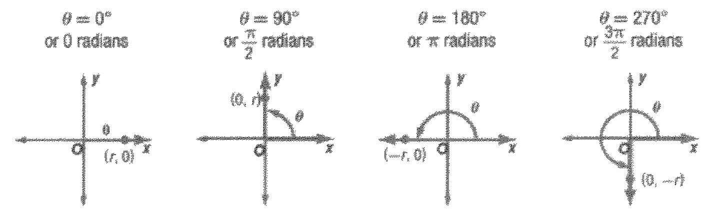
$\csc\theta = \frac{r}{y}, y \neq 0$ $\quad\sec\theta = \frac{r}{x}, x \neq 0$ $\quad\cot\theta = \frac{x}{y}, y \neq 0$



Quadrantal angle - an angle with a terminal side that lies on the x or y axis

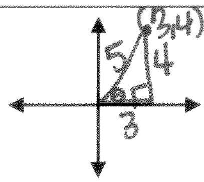


KeyConcept Quadrantal Angles



Examples: The terminal side of θ in standard position contains each point. Find the exact values of the six trigonometric functions of θ.

1. (3, 4)



$\sin\theta = \frac{4}{5}$

$\cos\theta = \frac{3}{5}$

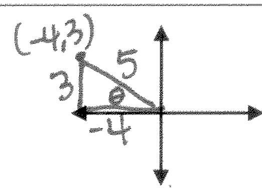
$\tan\theta = \frac{4}{3}$

$\csc\theta = \frac{5}{4}$

$\sec\theta = \frac{5}{3}$

$\cot\theta = \frac{3}{4}$

2. (-4, 3)



$\sin\theta = \frac{3}{5}$

$\cos\theta = -\frac{4}{5}$

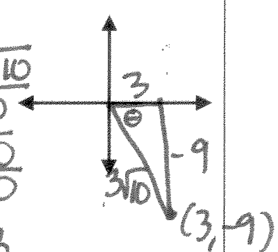
$\tan\theta = -\frac{3}{4}$

$\csc\theta = \frac{5}{3}$

$\sec\theta = -\frac{5}{4}$

$\cot\theta = -\frac{4}{3}$

3. (3, -9) $\quad\sqrt{9+81} = \sqrt{90} = 3\sqrt{10}$



$\sin\theta = \frac{-9}{3\sqrt{10}} = \frac{-3\sqrt{10}}{10}$

$\cos\theta = \frac{3}{3\sqrt{10}} = \frac{\sqrt{10}}{10}$

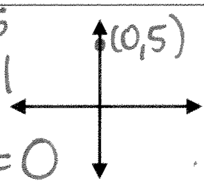
$\tan\theta = \frac{-9}{3} = -3$

$\csc\theta = \frac{-3\sqrt{10}}{9} = \frac{-\sqrt{10}}{3}$

$\sec\theta = \frac{3\sqrt{10}}{3} = \sqrt{10}$

$\cot\theta = -\frac{1}{3}$

4. (0, 5) $\quad r=5$



$\sin\theta = \frac{y}{r} = \frac{5}{5} = 1$

$\cos\theta = \frac{x}{r} = \frac{0}{5} = 0$

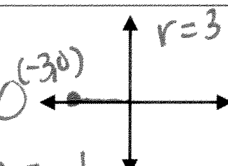
$\tan\theta = \frac{y}{x} = \frac{5}{0} = \varnothing$

$\csc\theta = \frac{r}{y} = \frac{5}{5} = 1$

$\sec\theta = \frac{r}{x} = \frac{5}{0} = \varnothing$

$\cot\theta = \frac{x}{y} = \frac{0}{5} = 0$

5. (-3, 0) $\quad r=3$



$\sin\theta = \frac{y}{r} = \frac{0}{3} = 0$

$\cos\theta = \frac{x}{r} = \frac{-3}{3} = -1$

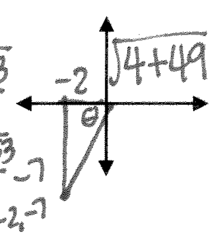
$\tan\theta = \frac{y}{x} = \frac{0}{-3} = 0$

$\csc\theta = \frac{1}{\sin\theta} = \frac{3}{0} = \varnothing$

$\sec\theta = \frac{1}{\cos\theta} = -1$

$\cot\theta = \frac{1}{\tan\theta} = \frac{-3}{0} = \varnothing$

6. (-2, -7) $\quad\sqrt{4+49} = \sqrt{53}$



$\sin\theta = \frac{-7}{\sqrt{53}} = \frac{-7\sqrt{53}}{53}$

$\cos\theta = \frac{-2}{\sqrt{53}} = \frac{-2\sqrt{53}}{53}$

$\tan\theta = \frac{-7}{-2} = \frac{7}{2}$

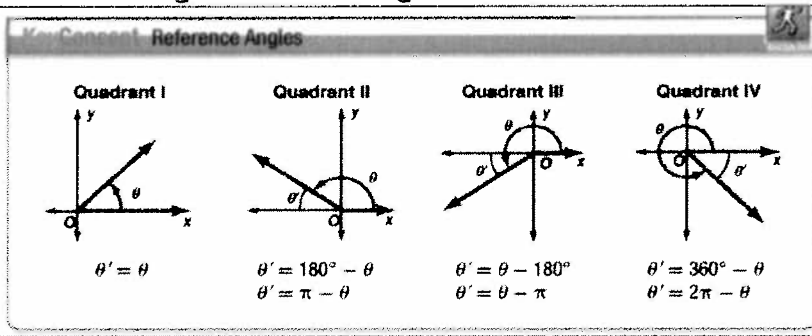
$\csc\theta = -\frac{\sqrt{53}}{7}$

$\sec\theta = -\frac{\sqrt{53}}{2}$

$\cot\theta = \frac{2}{7}$

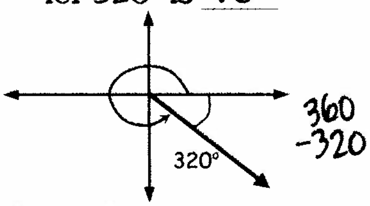
Reference angle - the acute angle formed by the terminal side of any angle and the x-axis.

The rules for finding a reference angle for $0° \le x \le 360°$ or $0° \le x \le 2\pi$ are below:

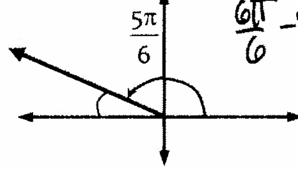


Examples: Find the reference angle of each angle.

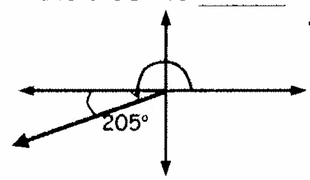
7. The reference angle for 320° is 40 °



8. The reference angle for $\frac{5\pi}{6}$ is $\frac{\pi}{6}$ $\quad \pi - \frac{5\pi}{6}$ $\quad \frac{6\pi}{6} - \frac{5\pi}{6}$

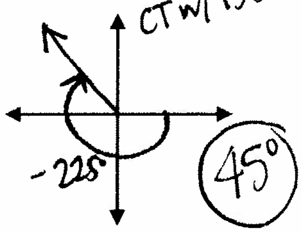


9. The reference angle for 205° is 25 ° $\quad$ 205 − 180 = 25

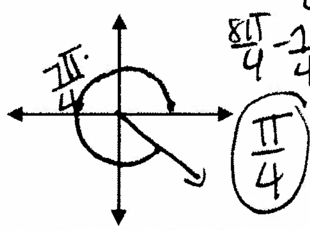


More Practice: Sketch each angle. Then find the reference angle.

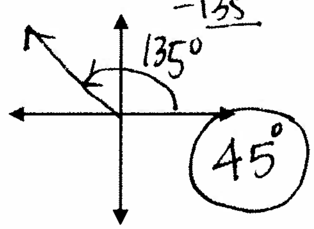
10. −225° CTW/ 135°



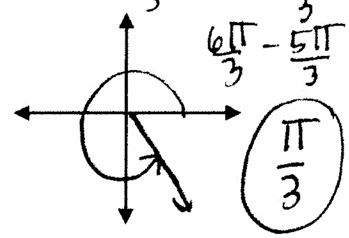
11. $\frac{7\pi}{4}$ $\quad 2\pi - \frac{7\pi}{4}$ $\quad \frac{8\pi}{4} - \frac{7\pi}{4}$



12. 135° $\quad$ 180 − 135

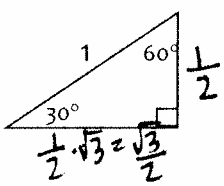


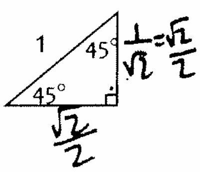
13. $\frac{5\pi}{3}$ $\quad 2\pi - \frac{5\pi}{3}$ $\quad \frac{6\pi}{3} - \frac{5\pi}{3}$



You can use reference angles of 30°, 45°, or 60° to evaluate trigonometric functions for any angle θ.

For HW problems 24-31 in the book work





Trigonometric Values for Special Angles					
Sine	Cosine	Tangent	Cosecant	Secant	Cotangent
$\sin 30° = \frac{1}{2}$	$\cos 30° = \frac{\sqrt{3}}{2}$	$\tan 30° = \frac{\sqrt{3}}{3}$	$\csc 30° = 2$	$\sec 30° = \frac{2\sqrt{3}}{3}$	$\cot 30° = \sqrt{3}$
$\sin 45° = \frac{\sqrt{2}}{2}$	$\cos 45° = \frac{\sqrt{2}}{2}$	$\tan 45° = 1$	$\csc 45° = \sqrt{2}$	$\sec 45° = \sqrt{2}$	$\cot 45° = 1$
$\sin 60° = \frac{\sqrt{3}}{2}$	$\cos 60° = \frac{1}{2}$	$\tan 60° = \sqrt{3}$	$\csc 60° = \frac{2\sqrt{3}}{3}$	$\sec 60° = 2$	$\cot 60° = \frac{\sqrt{3}}{3}$

KeyConcept Evaluate Trigonometric Functions

Step 1 Find the measure of the reference angle θ'.

Step 2 Evaluate the trigonometric function for θ'.

Step 3 Determine the sign of the trigonometric function value. Use the quadrant in which the terminal side of θ lies.

Quadrant II	Quadrant I
sin θ, csc θ: +	sin θ, csc θ: +
cos θ, sec θ: −	cos θ, sec θ: +
tan θ, cot θ: −	tan θ, cot θ: +
Quadrant III	**Quadrant IV**
sin θ, csc θ: −	sin θ, csc θ: −
cos θ, sec θ: −	cos θ, sec θ: +
tan θ, cot θ: +	tan θ, cot θ: −

Examples:

Q2 + a. sin 135° = $\frac{\sqrt{2}}{2}$
R.A. = 45°

Q1 + b. $\cot \frac{7\pi}{3}$ = $\frac{A}{O} = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{\sqrt{3}}{3}$
R.A. 60°

Q2 − c. cos 120° = $-\frac{1}{2}$
R.A. 60°