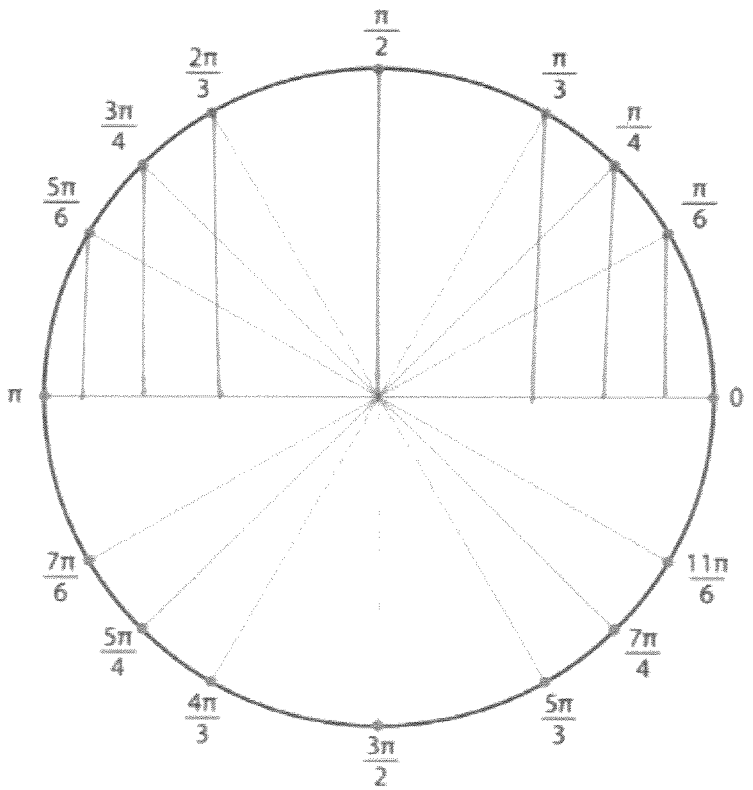
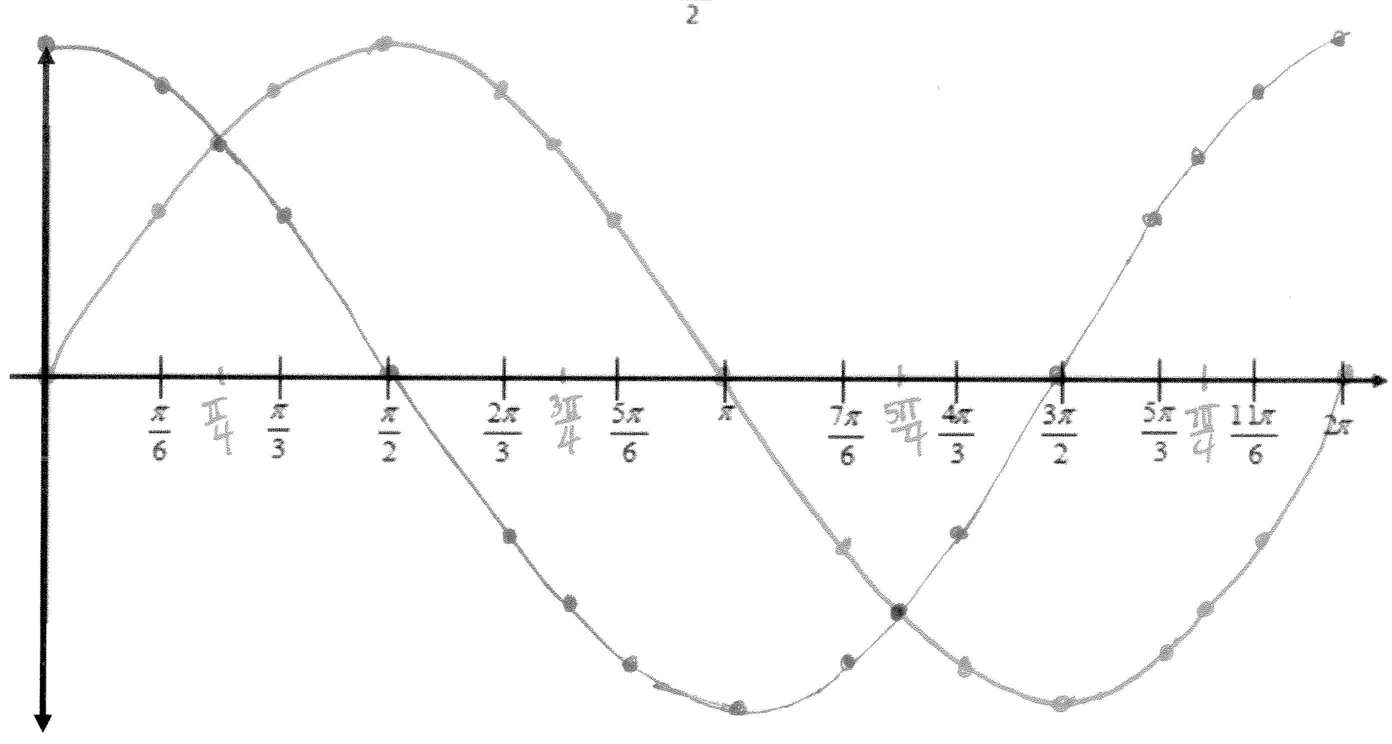
Master Ee

13-7 Graphing Trigonometric Functions

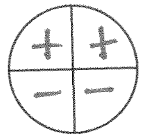
The Unit Circle - Radians

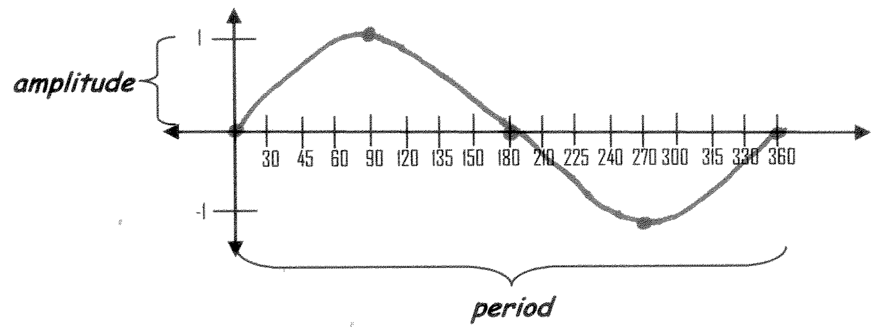




$f(x) = \sin x$ $\qquad$ $f(x) = \cos x$

Graphing y = sin x



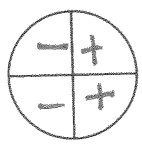


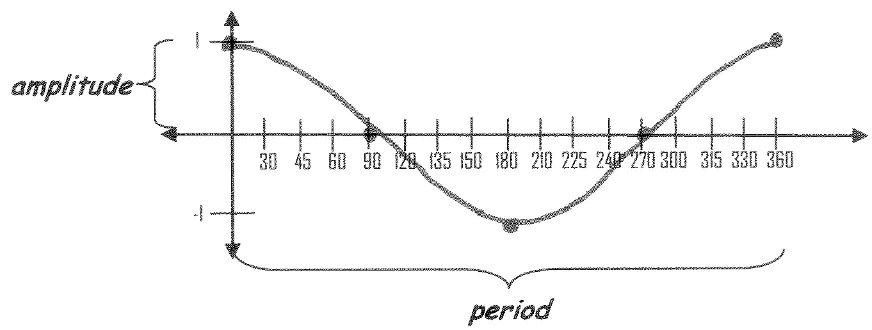
Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

		.5	.7	.9		.9	.7	.5		-.5	-.7	-.9		-.9	-.7	-.5	
θ	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
y = sin θ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0

Graphing y = cos x





Domain: $(-\infty, \infty)$

Range: $[-1, 1]$

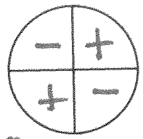
θ	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
y = cos θ	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{3}}{2}$	-1	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{2}}{2}$	$-\frac{1}{2}$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1

- In each function, **x** is the angle and **y** is the ratio
- Each function is **periodic** - the graph has a repeating pattern that continues to infinity!
- Each function has a **cycle** - the shortest repeating portion.
- Each function has a **period** - the horizontal length of each cycle.
- Each function has **amplitude** - the highest point of each graph from the x-axis.
- There will always be 4 **critical points**: the points that occur at $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{4}{4}$ of the period.
- The domain of each function is always all real numbers and the range is always $-1 \le y \le 1$!

Steps to graph y = a sin bx and y = a cos bx:

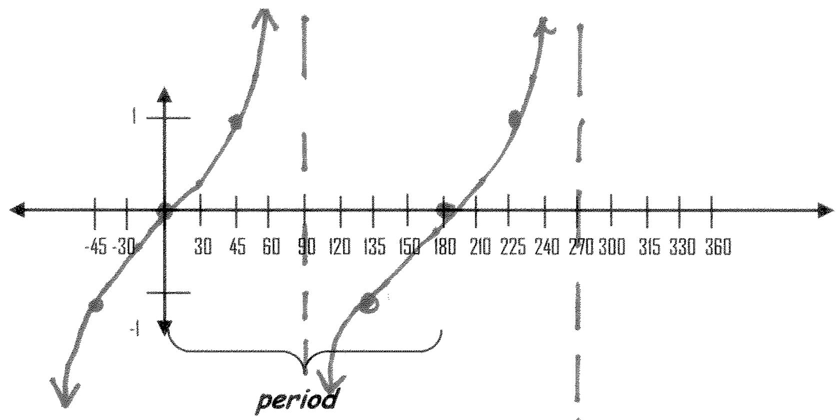
1. Calculate the period and the amplitude. **Period =** $\frac{360^\circ}{b}$ **or** $\frac{2\pi}{b}$ **Amplitude =** $|a|$
2. Break the x-axis into 4 equal parts. Write the degrees or radians under each tic mark.
   Each tic mark is ¼ of the period.
3. Make 2 tic marks on the y-axis (+a above the x-axis and –a below the x-axis)
4. Graph the 4 critical points, which are always the same! The only thing that will change are the labels!

Graphing y = tan x



Domain: $\mathbb{R}, x \neq 90 + 180n$

Range: $(-\infty, \infty)$



		-.6		.6		1.7		-1.7		-.6		.6		1.7	
θ	-45	-30	0	30	45	60	90	120	135	150	180	210	225	240	270
y = tan θ	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\varnothing$	$-\sqrt{3}$	-1	$-\frac{\sqrt{3}}{3}$	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	$\varnothing$

- There is no amplitude! The range is always all real numbers!
- The graph has a period of π.
- There are vertical asymptotes at odd multiples of $\frac{\pi}{2|b|}$
- The domain will always be all real numbers except for odd multiples of $\frac{\pi}{2}$

Formula: $\mathbb{R}, x \neq \frac{\pi}{2} + \pi n$

Steps to graph y = a tan bx:

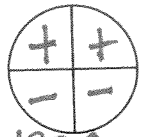
1. Calculate the period. **Period = $\frac{180^\circ}{b}$ or $\frac{\pi}{b}$**
2. Break the x-axis into 8 equal parts (2 to the left, and 6 to the right of the y-axis).
3. Write the degrees or radians under each tic mark. Each tic mark is ¼ of the period.
4. Make 2 tic marks on the y-axis (+a above the x-axis and –a below the x-axis)
5. Draw the asymptotes. The first will always be at $\frac{1}{2}$ of the period!
6. Plot the three critical points: the 1st (-a) at -¼ of the period, the 2nd (0) at the origin, and the 3rd (a) at ¼ of the period. Connect the three points like a cubic function. Draw one more branch to the right!

Writing the equation of a trigonometric function when given the graph:

1. Look to see how many cycles are drawn in one period. For Sine & Cosine, the period is $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$.

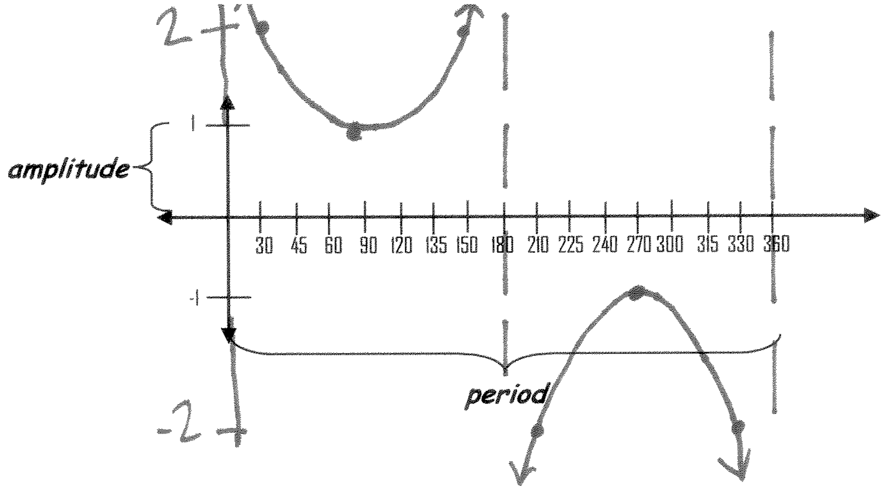
   For Tangent, the period is $\frac{180^\circ}{b}$ or $\frac{\pi}{b}$. Look at the # to the right of the graph and see how many periods equal that value. Solve for b using the period you found.
2. Look at the amplitude. What is a?
3. Plug a and b into correct equation (**y = a sin bx = a cos bx, y = a tan bx**).

Graphing y = csc x



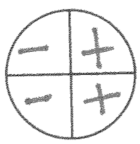
Domain: $\mathbb{R}, x \neq 180 + 180n$

Range: $(-\infty, -1] \cup [1, \infty)$



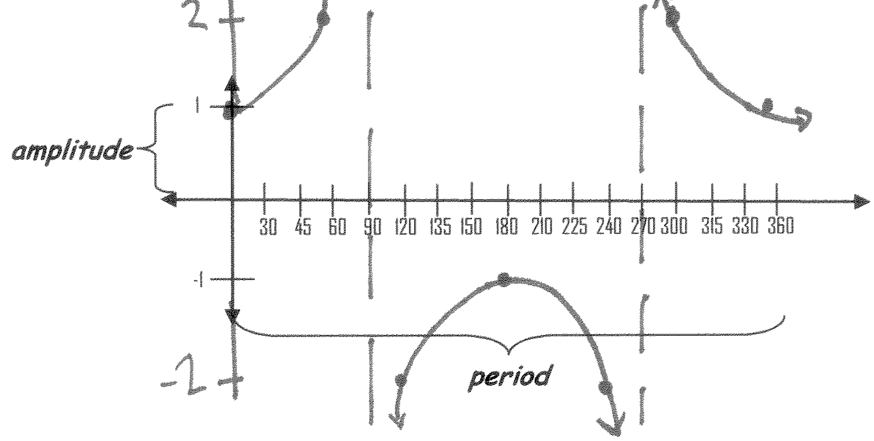
			1.4	1.2		1.2	1.4				-1.4	-1.2		-1.2	-1.4		
θ	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
y = csc θ	$\emptyset$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	$\emptyset$	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	$\emptyset$

Graphing y = sec x



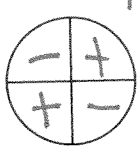
Domain: $\mathbb{R}, x \neq 90 + 180n$

Range: $(-\infty, \infty)$



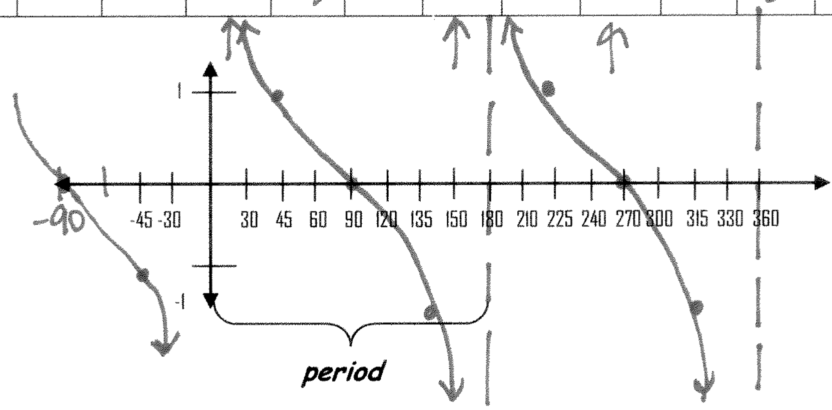
		1.2	1.4				-1.4	-1.2		-1.2	-1.4				1.4	1.2	
θ	0	30	45	60	90	120	135	150	180	210	225	240	270	300	315	330	360
y = sec θ	1	$\frac{2\sqrt{3}}{3}$	$\sqrt{2}$	2	$\emptyset$	-2	$-\sqrt{2}$	$-\frac{2\sqrt{3}}{3}$	-1	$-\frac{2\sqrt{3}}{3}$	$-\sqrt{2}$	-2	$\emptyset$	2	$\sqrt{2}$	$\frac{2\sqrt{3}}{3}$	1

Graphing y = cot x



Domain: $\mathbb{R}, x \neq 180 + 180n$

Range: $(-\infty, \infty)$



		-1.7		1.7		.6		-.6		-1.7		1.7		.6		
θ	-45	-30	0	30	45	60	90	120	135	150	180	210	225	240	270	315
y = cot θ	-1	$-\sqrt{3}$	$\emptyset$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	$-\frac{\sqrt{3}}{3}$	-1	$-\sqrt{3}$	$\emptyset$	$\sqrt{3}$	1	$\frac{\sqrt{3}}{3}$	0	-1