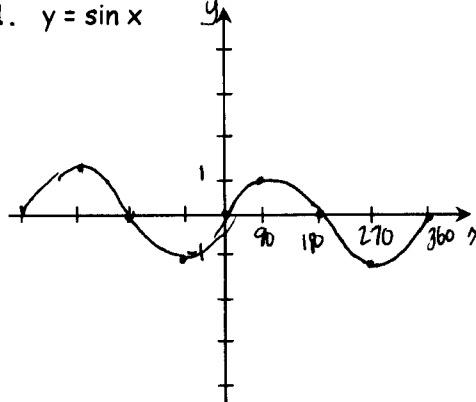
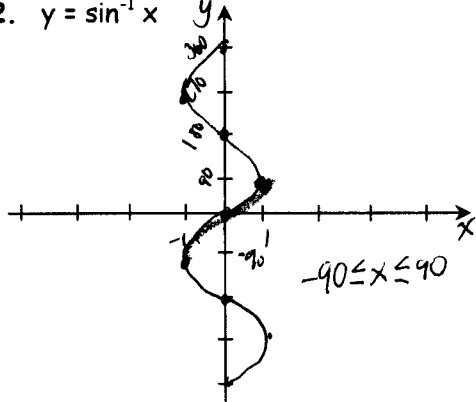
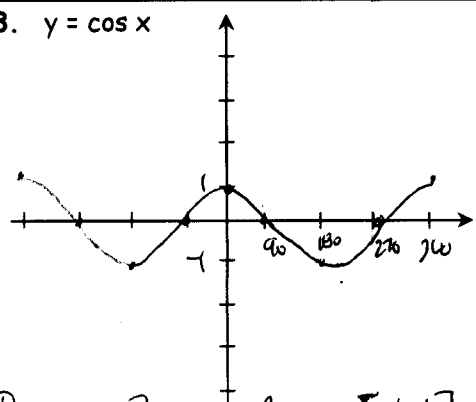
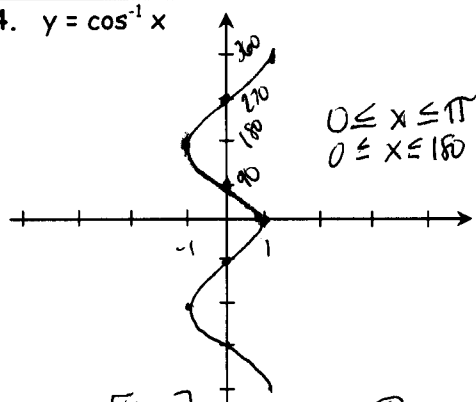
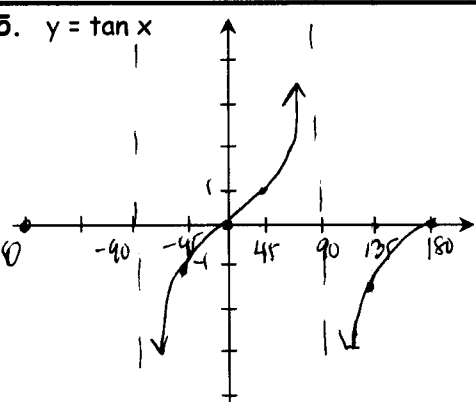
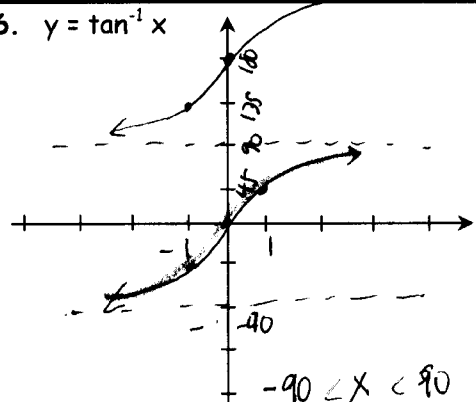


# 13-9 Inverse Trigonometric Functions

Name Master E  
Date \_\_\_\_\_ Block \_\_\_\_\_

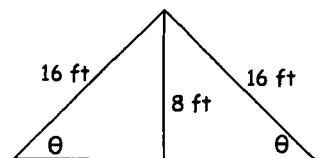
> Graph each of the following.

None of these are functions!

Trigonometric Function	Inverse Trigonometric Function (remember: switch x and y)																																		
<p>1. <math>y = \sin x</math></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>0</td></tr> <tr><td>90</td><td>1</td></tr> <tr><td>180</td><td>0</td></tr> <tr><td>270</td><td>-1</td></tr> <tr><td>360</td><td>0</td></tr> </table>	x	y	0	0	90	1	180	0	270	-1	360	0	<p>2. <math>y = \sin^{-1} x</math></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>90</td></tr> <tr><td>0</td><td>180</td></tr> <tr><td>-1</td><td>270</td></tr> <tr><td>0</td><td>360</td></tr> </table>	x	y	0	0	1	90	0	180	-1	270	0	360										
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<p>3. <math>y = \cos x</math></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>90</td><td>0</td></tr> <tr><td>180</td><td>-1</td></tr> <tr><td>270</td><td>0</td></tr> <tr><td>360</td><td>1</td></tr> </table> <p>Domain: <math>\mathbb{R}</math>      Range: <math>[-1, 1]</math></p>	x	y	0	1	90	0	180	-1	270	0	360	1	<p>4. <math>y = \cos^{-1} x</math></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>1</td><td>0</td></tr> <tr><td>0</td><td>90</td></tr> <tr><td>-1</td><td>180</td></tr> <tr><td>0</td><td>270</td></tr> <tr><td>1</td><td>360</td></tr> </table> <p>Domain: <math>[1, 1]</math>      Range: <math>\mathbb{R}</math></p>	x	y	1	0	0	90	-1	180	0	270	1	360										
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<p>5. <math>y = \tan x</math></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>0</td></tr> <tr><td>45</td><td>1</td></tr> <tr><td>90</td><td><math>\emptyset</math></td></tr> <tr><td>135</td><td>-1</td></tr> <tr><td>180</td><td>0</td></tr> <tr><td>225</td><td>1</td></tr> <tr><td>270</td><td><math>\emptyset</math></td></tr> <tr><td>315</td><td>-1</td></tr> <tr><td>360</td><td>0</td></tr> </table>	x	y	0	0	45	1	90	$\emptyset$	135	-1	180	0	225	1	270	$\emptyset$	315	-1	360	0	<p>6. <math>y = \tan^{-1} x</math></p>  <table border="1" style="margin-left: auto; margin-right: auto;"> <tr><th>x</th><th>y</th></tr> <tr><td>0</td><td>0</td></tr> <tr><td>1</td><td>45</td></tr> <tr><td>-1</td><td>135</td></tr> <tr><td>-1</td><td>-45</td></tr> <tr><td>0</td><td>180</td></tr> <tr><td><math>\emptyset</math></td><td></td></tr> </table>	x	y	0	0	1	45	-1	135	-1	-45	0	180	$\emptyset$	
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> The case for restrictions

If you have a tent with sides of 16 feet each and the middle pole of 8 feet, find the angle that the tent makes with the ground.



There are actually two angles whose  $\sin = \frac{1}{2}$ . What are they?  $30^\circ$  &  $150^\circ$

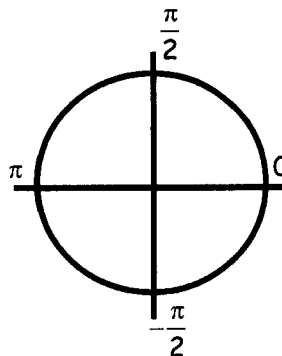
$$\sin \theta = \frac{8}{16} = \sin \theta = \frac{1}{2}$$

# Inverse Trigonometric Functions Practice

Name \_\_\_\_\_  
Date \_\_\_\_\_ Block \_\_\_\_\_

- > The inverse of a trigonometric function is not a function.\*
- > However, for functions with properly restricted domains, the inverse is a function.†
- > The values in these restricted domains are called principal values.
- > Capital letters are used to distinguish trigonometric functions with restricted domains from those with unrestricted domains.

Function	Restrictions	Inverse
$y = \cos x$	$0 \leq x \leq \pi$	$y = \cos^{-1} x$ or $y = \text{Arccos } x$
$y = \sin x$	$-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$	$y = \sin^{-1} x$ or $y = \text{Arcsin } x$
$y = \tan x$	$-\frac{\pi}{2} < x < \frac{\pi}{2}$	$y = \tan^{-1} x$ or $y = \text{Arctan } x$



\*\*The restricted domain of each function is the range of the inverse of that function.\*\*

> **Example:** Find  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ . Let  $\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ .

Then,  $\cos \theta = -\frac{\sqrt{3}}{2}$  ... For which angle in the restricted area is the  $\cos = \left(-\frac{\sqrt{3}}{2}\right)$ ?  $\rightarrow 150^\circ$

Therefore,  $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$

Find each value.

1.  $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right) = 60^\circ$

2.  $\cot^{-1} 1 = 45^\circ$

3.  $\text{Arctan}(-1) = -45^\circ$

4.  $\cos\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$   
 $\cos 45^\circ = \frac{\sqrt{2}}{2}$

5.  $\cot(\tan^{-1} 2) = \frac{1}{2}$

6.  $\sin\left(\text{Arcsin}\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{3}}{2}$

7.  $\tan\left(\text{Arcsin}\frac{5}{7}\right) = \frac{5}{2\sqrt{6}} = \frac{5\sqrt{6}}{12}$

8.  $\sin\left(\tan^{-1}\frac{5}{12}\right) = \frac{5}{13}$

9.  $\sin[\text{Arctan}(-\sqrt{2})] = \frac{-\sqrt{6}}{3}$

10.  $\text{Arccos}\left(-\frac{\sqrt{3}}{2}\right) = 150^\circ$

11.  $\text{Arcsin}\left(\frac{\sqrt{2}}{2}\right) = 45^\circ$

12.  $\text{Arccot}\left(-\frac{\sqrt{3}}{3}\right) = -60^\circ$