Master $\mathcal{E}$

7-1 OPERATIONS ON FUNCTIONS

Operations of Functions: Notation and Procedure

Sum:	Difference:	Product:	Quotient:
$(f+g)(x) =$ $f(x)+g(x)$	$(f-g)(x) =$ $f(x)-g(x)$	$(f \cdot g)(x) =$ $f(x)\cdot g(x)$	$\left(\frac{f}{g}\right)(x) =$ $f(x) \div g(x)$ where $g(x) \neq 0$

1-3: Perform the following operations for each problem.

1. $f(x) = 8x-3$; $g(x) = 4x+5$
   a. $(f+g)(x) = 8x-3+4x+5$
   $12x+2$
   b. $(f-g)(x) = 8x-3-(4x+5)$
   $8x-3-4x-5$
   $4x-8$
   c. $(f \cdot g)(x) = (8x-3)(4x+5)$
   $32x^2+40x-12x-15$
   $32x^2+28x-15$
   d. $\left(\frac{f}{g}\right)(x) = \frac{8x-3}{4x+5}, x \neq \frac{-5}{4}$

2. $f(x) = x^2+x-6$; $g(x) = x-2$
   a. $(f+g)(x) = x^2+x-6+x-2$
   x^2+2x-8
   b. $(f-g)(x) = x^2+x-6-(x-2)$
   $x^2+x-6-x+2$
   x^2-4
   c. $(f \cdot g)(x) = (x-2)(x^2+x-6)$
   $x^3+x^2-6x-2x^2-2x+12$
   $x^3-x^2-8x+12$
   d. $\left(\frac{f}{g}\right)(x) = \frac{x^2+x-6}{x-2}, x \neq 2$
   simp: $\frac{(x+3)(x-2)}{(x-2)} = x+3$!

3. $f(x) = 3x^2-x+5$; $g(x) = 2x-3$
   a. $(f+g)(x) = 3x^2-x+5+2x-3$
   $3x^2+x+2$
   b. $(f-g)(x) = 3x^2-x+5-(2x-3)$
   $3x^2-x+5-2x+3$
   $3x^2-3x+8$
   c. $(f \cdot g)(x) = (3x^2-x+5)(2x-3)$
   $6x^3-2x^2+10x-9x^2+3x-15$
   $6x^3-11x^2+13x-15$
   d. $\left(\frac{f}{g}\right)(x) = \frac{3x^2-x+5}{2x-3}, x \neq \frac{3}{2}$

Composition of functions

- the process of using the output of one function as the input of another function.
  The results where evaluating a value of one function is used to evaluate a value of a second function.
- **Composition of f and g:** written in the form $(f \circ g)(x) = f(g(x))$

4-11: Given $f(x) = \frac{3}{4}x - 3$ **and** $g(x) = |x|$**, find each value algebraically and then check it on the graph.**

4. $f(g(4))$ $g(4)=4$
   $f(4)=\frac{3}{4}(4)-3=0$

5. $g(f(4))$ $f(4)=0$
   $g(0)=0$

6. $f(g(2))$ $g(2)=2$
   $f(2)=\frac{3}{4}(2)-3=\frac{3}{2}-\frac{6}{2}=-\frac{3}{2}$

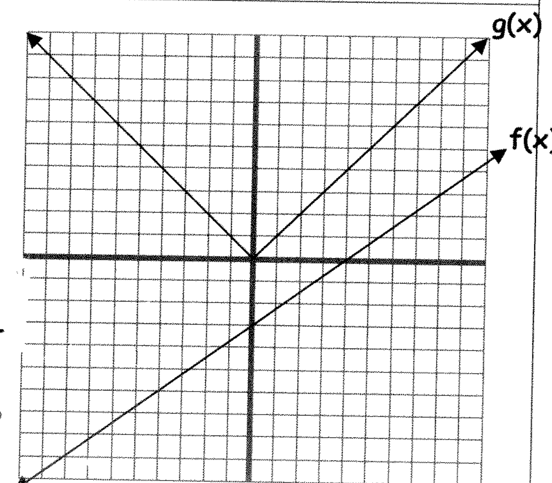
7. $g(f(2))$ $f(2)=-\frac{3}{2}$
   $g(-\frac{3}{2})=\frac{3}{2}$

8. $f(g(-2))$ $g(-2)=2$
   $f(2)=-\frac{3}{2}$ (#6)

9. $g(f(-2))$ $f(-2)=$
   $\frac{3}{4}(-2)-3=-\frac{3}{2}-\frac{6}{2}=-\frac{9}{2}$
   $g(-\frac{9}{2})=\frac{9}{2}$

10. $f(g(-4))$ $g(-4)=4$
    $f(4)=0$ (#4)

11. $g(f(-4))$ $f(-4)=$
    $\frac{3}{4}(-4)-3=-3-3=-6$
    $g(-6)=6$



Example 1 **For $f = \{(1, 2), (3, 3), (2, 4), (4, 1)\}$ and $g = \{(1, 3), (3, 4), (2, 2), (4, 1)\}$, find $f \circ g$ and $g \circ f$ if they exist.**

$f[g(1)] = f(3) = 3$ $f[g(2)] = f(2) = 4$ $f[g(3)] = f(4) = 1$ $f[g(4)] = f(1) = 2$,

So $f \circ g = \{(1, 3), (2, 4), (3, 1), (4, 2)\}$

$g[f(1)] = g(2) = 2$ $g[f(2)] = g(4) = 1$ $g[f(3)] = g(3) = 4$ $g[f(4)] = g(1) = 3$,

So $g \circ f = \{(1, 2), (2, 1), (3, 4), (4, 3)\}$

12-14: For each pair of functions, find f ∘ g and g ∘ f, if they exist.

12. $f = \{(-1, 2), (5, 6), (0, 9)\}$
$g = \{(6,0), (2, -1), (9, 5)\}$

$f \circ g = f(g(6)) = f(0) = 9$
$f(g(2)) = f(-1) = 2$
$f(g(9)) = f(5) = 6$

$f \circ g = \{(6,9), (2,2), (9,6)\}$

$g \circ f = g(f(-1)) = g(2) = -1$
$g(f(5)) = g(6) = 0$
$g(f(0)) = g(9) = 5$

$g \circ f = \{(-1,-1), (5,0), (0,5)\}$

13. $f = \{(-4, 3), (0, -2), (1, -2)\}$
$g = \{(-2,0), (3, 1)\}$

$f \circ g = f(g(-2)) = f(0) = -2$
$f(g(3)) = f(1) = -2$

$f \circ g = \{(-2,-2), (3,-2)\}$

$g \circ f = g(f(-4)) = g(3) = 1$
$g(f(0)) = g(-2) = 0$
$g(f(1)) = g(-2) = 0$

$g \circ f = \{(-4,1), (0,0), (1,0)\}$

14. $f = \{(-5, 4), (14, 8), (12, 1), (0, -3)\}$
$g = \{(-2, -4), (-3, 2), (-1, 4), (5, -6)\}$

$f \circ g = f(g(-2)) = f(-4) = \varnothing$
$f(g(-3)) = f(2) = \varnothing$
$f(g(-1)) = f(4) = \varnothing$
$f(g(5)) = f(-6) = \varnothing$

$f \circ g$ doesn't exist

$g \circ f = g(f(-5)) = g(4) = \varnothing$
$g(f(14)) = g(8) = \varnothing$
$g(f(12)) = g(1) = \varnothing$
$g(f(0)) = g(-3) = 2$

$g \circ f = \{(0,2)\}$

15-16: Find [f ° g](x) and [g ° f](x), if they exist.

15. $f(x) = 2x + 7$; $g(x) = -5x - 1$

$f(g(x)) = f(-5x-1) = 2(-5x-1)+7$
$= -10x-2+7$

$[f \circ g](x) = -10x+5$

$g(f(x)) = g(2x+7) = -5(2x+7)-1$
$= -10x-35-1$

$[g \circ f](x) = -10x-36$

16. $f(x) = x^2 + 2x$; $g(x) = x - 9$

$f(g(x)) = f(x-9) = (x-9)^2 + 2(x-9)$
$= x^2 - 18x + 81 + 2x - 18$

$[f \circ g](x) = x^2 - 16x + 63$

$g(f(x)) = g(x^2+2x) = x^2 + 2x - 9$

$[g \circ f](x) = x^2 + 2x - 9$