7-2 Inverse Functions and Relations

Name Master Copy
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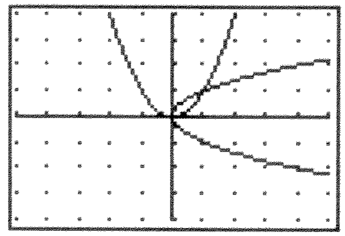
RECALL: Definition of a **relation** – a set of ordered pairs.

Definition of a **function** – a relation in which each element of the domain is paired with one and only one element of the range . . . IN OTHER WORDS - a relation in which the x-coordinates are NOT repeated. (Think of the vertical line test.)

INVERSE of a FUNCTION:
- is always a relation,
- may or may not be a function.

Example: Relation: {(3, 8), (4, -2), (5, -3)}
Inverse: {(8, 3), (-2, 4), (-3, 5)}



Use the vertical line test.

Two relations are inverse relations if and only if whenever one relation contains the element (a,b), the other relation contains the element (b,a).

To determine whether the inverse of a function is itself a function: use the HORIZONTAL line test.
*If no horizontal line intersects the graph of a function *f* more than once, then the ***INVERSE*** of *f* is itself a function.

*If f and f^{-1} are inverses, then f(a) = b if and only if f^{-1}(b) = a.

To graph the inverse of a function:
Example: $f(x) = x^2 + 1$

Use a table of values.

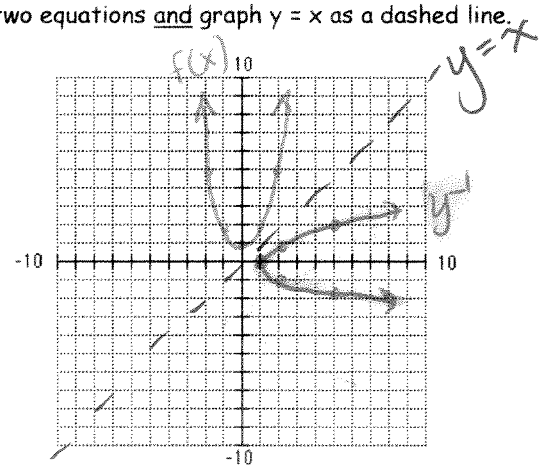
Then, find the inverse by switching the order of the ordered pairs.

x	y
-2	5
-1	2
0	1
1	2
2	5

→

x	y
5	-2
2	-1
1	0
2	1
5	2

Now graph each set of points to get the graphs of the two equations and graph y = x as a dashed line.



To find the inverse of a function:
1. Change f(x) to y.
2. Switch the x's and y's.
3. Solve for y.
4. Change y to $f^{-1}(x)$. (. . . $f^{-1}(x)$ is read "f inverse of x".)

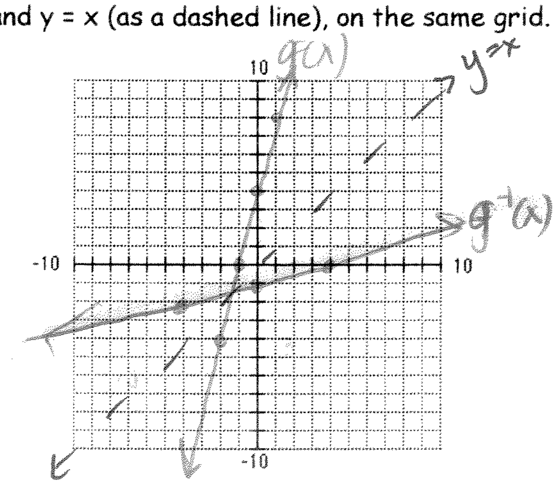
Example: g(x) = 4x + 4

$y = 4x + 4$
$x = 4y + 4$
$x - 4 = 4y$

inverse function notation

$g^{-1}(x) = \frac{1}{4}x - 1$

Now graph each function (g(x) and $g^{-1}(x)$), and y = x (as a dashed line), on the same grid.



Find the inverse of the given function.

1.

x	1	2	3	4	5
y	-6	-3	0	3	6

$\{(-6,1)(-3,2)(0,3)(3,4)(6,5)\}$

2. {(7, 7), (4, 9), (3,-2)}

$\{(7,7),(9,4)(-2,3)\}$

Inverse Functions: two function, f and g, are inverse functions if and only if both their compositions are the identity function. That is, $f(g(x)) = g(f(x)) = x$.

Verify that f and g are inverse functions by showing that f(g(x)) = g(f(x)) = x. Graph on TI to show inverse relationship

3. $f(x) = 1 - x$, $g(x) = 1 - x$

$f(g(x)) = 1-(1-x) = 1-1+x = x$

$g(f(x)) = 1-(1-x) = 1-1+x = x$

4. $f(x) = -3x + 6$, $g(x) = -\frac{1}{3}x + 2$

$f(g(x)) = -3(-\frac{1}{3}x+2)+6 = x-6+6 = x$

$g(f(x)) = -\frac{1}{3}(-3x+6)+2 = x-2+2 = x$

5. f(g(2)) = 2 g(f(2)) = 2

6. f(g(2)) = 2 g(f(2)) = 2

Find the equation for the inverse of each function.

7. $f(x) = 4x - 9$

$x = 4y-9$

$x+9 = 4y$

$\frac{1}{4}x + \frac{9}{4} = y$

$f^{-1}(x) = \frac{1}{4}x + \frac{9}{4}$

8. $g(x) = x^2 - 9$

$x = y^2 - 9$

$x+9 = y^2$

$\pm\sqrt{x+9} = y$

$y^{-1} = \pm\sqrt{x+9}$

9. $h(x) = \frac{1}{2}x + 6$

$x = \frac{1}{2}y + 6$

$x - 6 = \frac{1}{2}y$

$2x - 12 = y$

$h^{-1}(x) = 2x - 12$

10. $f(x) = \frac{1}{x}$

$x = \frac{1}{y}$

$xy = 1$

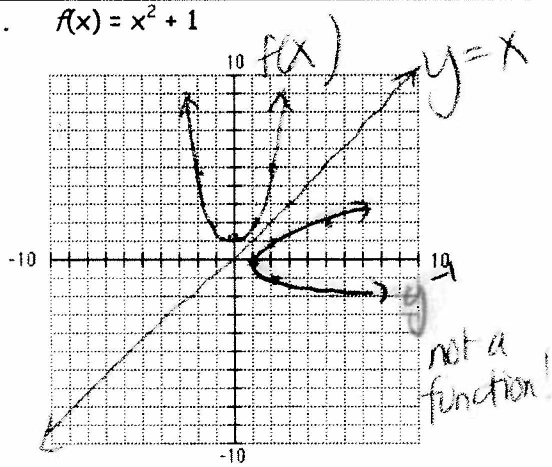
$y = \frac{1}{x}$

$f^{-1}(x) = \frac{1}{x}$

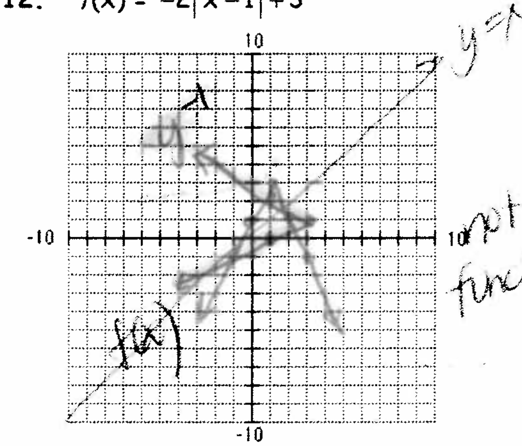
The recip f(x) is its own inverse

Graph the function, its inverse, and the line of reflection. State whether the inverse of the function is itself a function.

11. $f(x) = x^2 + 1$



12. $f(x) = -2|x-1|+3$



not a function