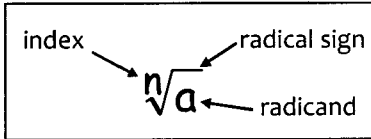


7-4 & 7-5 nth Roots & Operations with Radical Expressions

Master E

Simplify Radicals: Finding the square root of a number and squaring a number are inverse operations. Also, the inverse of raising a number to the nth power is finding the nth root of a number.

The nth Root of a Number: To solve a square root you must find the number that, when multiplied by itself, results in the radicand. To solve an nth root you must find the number that, when multiplied by itself n times, results in the radicand.



Note: When there is no index shown, the index is understood to be 2 (square root)

Suppose n is an integer greater than 1 and a is a real number. The following will be true.

a	If n is even:	If n is odd:
a > 0	There are 2 real roots: $\sqrt[n]{a} = \pm 2$	There is one real positive root: $\sqrt[5]{32} = 2$
a < 0	There are no real roots: $\sqrt[4]{-16} = \text{no real roots}$	There is one real negative root: $\sqrt[3]{-64} = -4$
a = 0	This is one root: $\sqrt[7]{0} = 0$	This is one root: $\sqrt[9]{0} = 0$

Principal Root: Some numbers have more than one real nth root. For example, 64 has 2 square roots, ± 8 , since 8^2 and $(-8)^2$ both equal 64. When n is even, there is more than one real root; the positive root is called the **principal root**.

Formulas: $\sqrt[n]{a^n} = a$ $\sqrt[n]{a^{2n}} = a^2$ $\sqrt[n]{a^{3n}} = a^3$ $\sqrt[n]{a^m} = a^{\frac{m}{n}}$ $\sqrt[n]{(a+b)^n} = a+b$

1-14: Simplify each expression. Pay attention to the details... signs DO matter!

1. $\sqrt{36}$ 6	2. $\pm\sqrt{36}$ ± 6	3. $-\sqrt{36}$ -6	4. $\sqrt{-36}$ $\pm 6i$	5. $-\sqrt{-36}$ $\pm 6i$	6. $\sqrt[3]{64}$ 4	7. $-\sqrt[3]{64}$ -4
8. $\sqrt[3]{-64}$ -4	9. $-\sqrt[3]{-64}$ 4	10. $\sqrt[4]{81}$ 3	11. $-\sqrt[4]{81}$ -3	12. $\sqrt[4]{-81}$ imag. sol.	13. $\sqrt[4]{x^8}$ x^2	14. $\sqrt[3]{x^{12}}$ x^4

Approximate nth roots with a calculator: Example: $\sqrt[5]{32}$

Option 1: Use the MATH menu and the $\sqrt[n]{}$ function. First input n, then $\sqrt[n]{}$, then the radicand, then ENTER. (Keystrokes: 5 → MATH → $\sqrt[n]{}$ → 32 → ENTER = 2)

Option 2: Enter $32^{(1 \div 5)}$ → ENTER

15-18: Use a calculator to approximate each value to three decimal places.

15. $\sqrt{1050}$ 32.404	16. $\sqrt[3]{-15}$ -2.466	17. $\sqrt[5]{100}$ 2.5119	18. $-\sqrt[4]{500}$ -4.729
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19-22: Write each expression in simplified radical form.

19. $\sqrt{x^2 - 4x + 4}$ $\sqrt{(x-2)^2}$ x-2	20. $\sqrt[3]{-(y-9)^9}$ $-(y-9)^3$	21. $\sqrt[8]{x^{16}y^8} \sqrt[8]{x^{16}y^8}$ $\sqrt{x^{32}y^{16}}$ $x^4 y^2$	22. $\pm\sqrt{49x^4}$ $\pm 7x^2$
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Simplest Form of a Radical: For a radical to be in simplest form, it must meet the following conditions:

- The index n is as small as possible
- The radicand contains no factors other than 1 that are nth powers of an integer or polynomial
- The radicand contains no fractions
- No radicals appear in a denominator

23-26: Simplify each radical completely.

23. $\sqrt{12c^6d^3}$ $\sqrt{4 \cdot 3c^6d^2d}$ $2c^3d\sqrt{3d}$	24. $\sqrt[3]{27x^{12}z^7}$ $3^3\sqrt{y^{12}z^6z}$ $3y^4z^2\sqrt[3]{z}$	25. $\sqrt{50a^5b^9}$ $\sqrt{25 \cdot 2 \cdot a^4 \cdot b^8 \cdot b}$ $5a^2b^4\sqrt{2b}$	26. $\sqrt[3]{250m^{30}p^{20}}$ $3\sqrt{125 \cdot 2 \cdot m^{30} \cdot p^{18} \cdot p^2}$ $5m^{10}p^6\sqrt[3]{2p^2}$
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Multiplying Radicals: You must use the Product Property.

PRODUCT PROPERTY: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$ <i>n can be odd or even (a & b must be positive)</i>	Examples: a. $a.\sqrt[3]{25} \cdot \sqrt[3]{5} = \sqrt[3]{25 \cdot 5} = \sqrt[3]{125} = 5$ $b.\sqrt[4]{64} = \sqrt[4]{16 \cdot 4} = \sqrt[4]{16} \cdot \sqrt[4]{4} = 2\sqrt[4]{4}$
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27-28: Simplify each radical completely.

27. $6\sqrt{8c^3d^5} \cdot 4\sqrt{2cd^3}$ $24\sqrt{16c^4d^8}$ $24 \cdot 4 \cdot c^2 \cdot d^4$ $96c^2d^4$	28. $5\sqrt[3]{100a^2} \cdot \sqrt[3]{10a}$ $5\sqrt[3]{1000a^3}$ $5 \cdot 10a = 50a$
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Dividing Radicals: You must use the Quotient Property or Rationalize the Denominator when it doesn't reduce.

QUOTIENT PROPERTY: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ <i>n must be positive and b can't equal zero</i>	Examples: $a.\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$ $b.\sqrt[4]{\frac{7}{8}} = \frac{\sqrt[4]{7}}{\sqrt[4]{8}} = \frac{\sqrt[4]{7}}{\sqrt[4]{2^3}} \cdot \frac{\sqrt[4]{2}}{\sqrt[4]{2}} = \frac{\sqrt[4]{7 \cdot 2}}{\sqrt[4]{2^4}} = \frac{\sqrt[4]{14}}{2}$
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How to Rationalize the Denominator

If the denominator is:	Multiply the numerator and denominator by:	Examples:
\sqrt{b}	\sqrt{b}	$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{6}{\sqrt[4]{2}} = \frac{6}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \frac{6\sqrt[4]{8}}{2} = 3\sqrt[4]{8}$
$a\sqrt{b} + c\sqrt{d}$ Or $a\sqrt{b} - c\sqrt{d}$	$a\sqrt{b} - c\sqrt{d}$ Or $a\sqrt{b} + c\sqrt{d}$	$\frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}} = \frac{\sqrt{2}}{\sqrt{5} - \sqrt{3}} \cdot \frac{(\sqrt{5} + \sqrt{3})}{(\sqrt{5} + \sqrt{3})} = \frac{\sqrt{10} + \sqrt{6}}{5 + \sqrt{15} - \sqrt{15} - 3} = \frac{\sqrt{10} + \sqrt{6}}{2}$

29-32: Simplify each radical completely.

29. $\frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{30}}{2}$	30. $\frac{\sqrt{y^8} \sqrt{x}}{\sqrt{x^2} \sqrt{x}} = \frac{\sqrt{xy^8}}{\sqrt{x^3}} = \frac{y^4\sqrt{x}}{x^3}$	31. $\frac{\sqrt{2} \cdot \sqrt[3]{2} \cdot \sqrt[3]{3x^2}}{\sqrt{9x} \cdot \sqrt[3]{3^2x} \cdot \sqrt[3]{3x^2}} = \frac{\sqrt[3]{6x^2}}{\sqrt[3]{3^3x^3}} = \frac{\sqrt[3]{6x^2}}{3x}$	32. $\sqrt[4]{\frac{3}{4y}} \cdot \frac{\sqrt[4]{3}}{\sqrt[4]{2^2y}} \cdot \frac{\sqrt[4]{2^2y^3}}{\sqrt[4]{2^2y^3}} = \frac{\sqrt[4]{12y^3}}{2y}$
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Adding & Subtracting Radicals: You must have like terms to add or subtract radicals!

Two radicals are like radicals if they have the same index and the same radicand. Use the distributive property to add or subtract like radicals	Example: $5\sqrt[4]{x^3} - 3\sqrt[4]{x^3} = (5-3)\sqrt[4]{x^3} = 2\sqrt[4]{x^3}$
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33-34: Simplify each radical completely.

33. $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$ $3 \cdot 3\sqrt{5} - 5 \cdot 4\sqrt{5} + 4 \cdot 2\sqrt{5}$ $9\sqrt{5} - 20\sqrt{5} + 8\sqrt{5}$ $\sqrt{5}(9-20+8) = -3\sqrt{5}$	34. $9\sqrt[3]{16} + 3\sqrt[3]{2} - \sqrt[3]{128}$ $9\sqrt[3]{8 \cdot 2} + 3\sqrt[3]{2} - \sqrt[3]{64 \cdot 2}$ $9 \cdot 2\sqrt[3]{2} + 3\sqrt[3]{2} - 4\sqrt[3]{2}$ $18\sqrt[3]{2} + 3\sqrt[3]{2} - 4\sqrt[3]{2}$ $\sqrt[3]{2}(18+3-4) = 17\sqrt[3]{2}$
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