8-1 Geometric Mean

Geometric Mean The **geometric mean** between two numbers is the positive square root of their product. For two positive numbers a and b, the geometric mean of a and b is the positive number x in the proportion $\frac{a}{x} = \frac{x}{b}$. Cross multiplying gives $x^2 = ab$, so $x = \sqrt{ab}$.

Example **Find the geometric mean between each pair of numbers.**

a. 12 and 3

$x = \sqrt{ab}$	Definition of geometric mean
$= \sqrt{12 \cdot 3}$	$a = 12$ and $b = 3$
$= \sqrt{(2 \cdot 2 \cdot 3) \cdot 3}$	Factor.
$= 6$	Simplify.

The geometric mean between 12 and 3 is 6.

b. 8 and 4

$x = \sqrt{ab}$	Definition of geometric mean
$= \sqrt{8 \cdot 4}$	$a = 8$ and $b = 4$
$= \sqrt{(2 \cdot 4) \cdot 4}$	Factor.
$= \sqrt{16 \cdot 2}$	Associative Property
$= 4\sqrt{2}$	Simplify.

The geometric mean between 8 and 4 is $4\sqrt{2}$ or about 5.7.

Exercises: Find the geometric mean between each pair of numbers.

1. 4 and 4 4

2. 4 and 6 $2\sqrt{6}$

3. 6 and 9 $3\sqrt{6}$

4. $\frac{1}{2}$ and 2 1

5. 12 and 20 $\sqrt{4 \cdot 3 \cdot 4 \cdot 5}$ $4\sqrt{15}$

6. 4 and 25 10

7. 16 and 30 $4\sqrt{30}$

8. 10 and 100 $10\sqrt{10}$

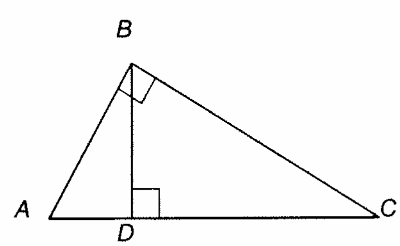
9. $\frac{1}{2}$ and $\frac{1}{4}$ $\frac{1}{\sqrt{8}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

10. 17 and 3 $\sqrt{51}$

11. 4 and 16 8

12. 3 and 24 $\sqrt{3 \cdot 4 \cdot 6 \cdot 2 \cdot 3}$ $6\sqrt{2}$

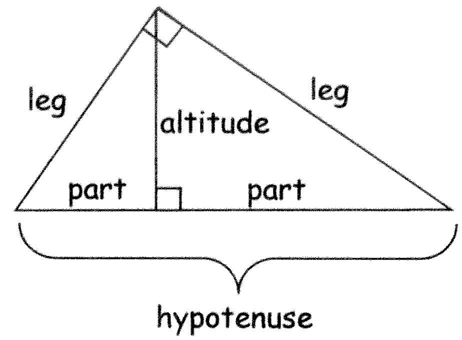
Geometric Means in Right Triangles In the diagram, $\triangle ABC \sim \triangle ADB \sim \triangle BDC$. An altitude to the hypotenuse of a right triangle forms two right triangles. The two triangles are similar and each is similar to the original triangle.



Summary of Formulas:

$$\frac{\text{part}}{\text{leg}} = \frac{\text{leg}}{\text{hypotenuse (whole)}} \qquad \text{Leg}^2 = \text{Part} \cdot \text{Whole}$$

$$\frac{\text{part}}{\text{altitude}} = \frac{\text{altitude}}{\text{part}} \qquad \text{Altitude}^2 = \text{Part} \cdot \text{Part}$$



Example 1 **Use right $\triangle ABC$ with $\overline{BD} \perp \overline{AC}$. Describe two geometric means.**

a. $\triangle ADB \sim \triangle BDC$ so $\frac{AD}{BD} = \frac{BD}{CD}$.

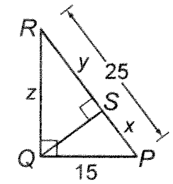
In $\triangle ABC$, the altitude is the geometric mean between the two segments of the hypotenuse.

b. $\triangle ABC \sim \triangle ADB$ and $\triangle ABC \sim \triangle BDC$,

so $\frac{AC}{AB} = \frac{AB}{AD}$ and $\frac{AC}{BC} = \frac{BC}{DC}$.

In $\triangle ABC$, each leg is the geometric mean between the hypotenuse and the segment of the hypotenuse adjacent to that leg.

Example 2 **Find x, y, and z.**



$15 = \sqrt{RP \cdot SP}$ Geometric Mean (Leg) Theorem

$15 = \sqrt{25x}$ $RP = 25$ and $SP = x$

$225 = 25x$ Square each side.

$9 = x$ Divide each side by 25.

Then

$y = RP - SP$

$= 25 - 9$

$= 16$

$z = \sqrt{RS \cdot RP}$ Geometric Mean (Leg) Theorem

$= \sqrt{16 \cdot 25}$ $RS = 16$ and $RP = 25$

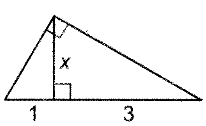
$= \sqrt{400}$ Multiply.

$= 20$ Simplify.

Exercises

Find x, y, and z to the nearest tenth.

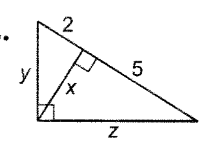
1.



$\frac{1}{x} = \frac{x}{3}$ $x^2 = 3$

$x = \sqrt{3}$

2.

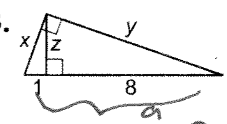


$\frac{2}{x} = \frac{x}{5}$ $x = \sqrt{10}$

$\frac{2}{y} = \frac{y}{7}$ $y = \sqrt{14}$

$\frac{5}{z} = \frac{z}{7}$ $z = \sqrt{35}$

3.

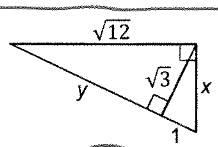


$\frac{1}{x} = \frac{x}{9}$ $x = 3$

$\frac{8}{y} = \frac{y}{9}$ $y = \sqrt{4 \cdot 2 \cdot 9}$ $6\sqrt{2}$

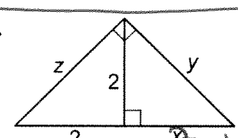
$\frac{1}{z} = \frac{z}{8}$ $2\sqrt{2}$

4.



$x = 2$ $y = 3$

5.



$\frac{2}{z} = \frac{z}{2+x}$ 4

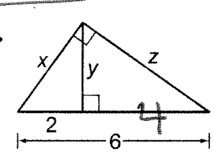
$z^2 = 2 \cdot 4$

$z = 2\sqrt{2}$

$\frac{2}{2} = \frac{2}{x}$ $2x = 4$ $x = 2$

$\frac{2}{y} = \frac{y}{4}$ $y = 2\sqrt{2}$

6.



$\frac{2}{x} = \frac{x}{6}$, $\frac{2}{y} = \frac{y}{4}$ $\frac{4}{z} = \frac{z}{6}$

$x^2 = 12$ $y^2 = 2 \cdot 4$

$x = 2\sqrt{3}$ $y = 2\sqrt{2}$ $2\sqrt{6} = z$