8-3 Logarithms & Graphing Logarithmic Functions

*********** **COMMIT THIS TO MEMORY: "A logarithm is an exponent!!"** ***********

Using the algebraic method, find the inverse of the exponential equation: $f(x) = 2^x$

STEP 1: Change f(x) to y. $y = 2^x$

STEP 2: Switch x and y $x = 2^y$

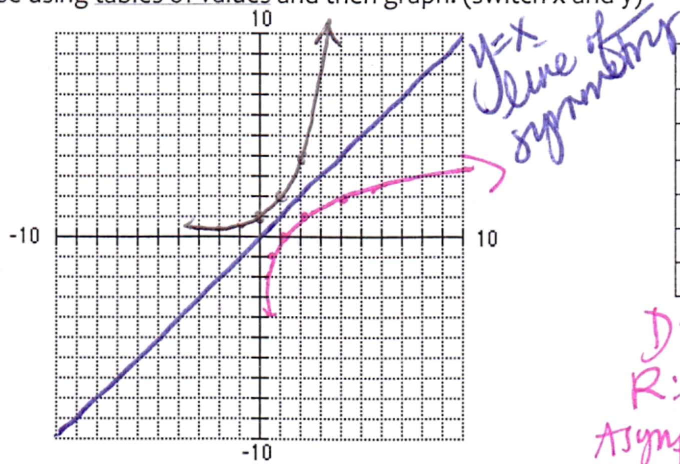
STEP 3: Solve for y. ______ We get stuck at this point. Why ? . . . .

STEP 4: Write y as $f^{-1}(x)$ ______

So, try finding the inverse using tables of values and then graph. (switch x and y)

$y = 2^x$	
x	y
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4

D: $(-\infty, \infty)$
R: $(0, \infty)$
Asym: $y = 0$



inverse	
x	y
$\frac{1}{4}$	-2
$\frac{1}{2}$	-1
1	0
2	1
4	2

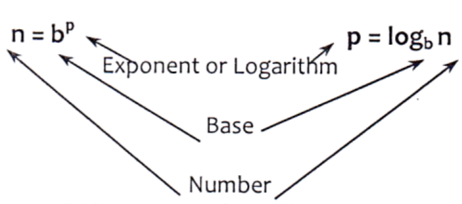
D: $(0, \infty)$
R: $(-\infty, \infty)$
Asymp: $x = 0$

Since we can graph it, there must be an equation for it. As we see in steps 3 and 4 above, to this point we do not have the notation necessary to write the equation of the inverse of an exponential function. To address this issue new notation was developed. Here's an example of how the notation was developed . . .

- If $f(x) = 2^x$ then ⟶ $f(3) = 2^3$
  and $f(3) = 8$. The inverse could be written as $f^{-1}(8) = 3$ (Switch the x and y).
- The notation for this was changed to $\mathbf{f_2(8) = 3}$ and then $\mathbf{pwr_2(8) = 3}$ and $\mathbf{exp_2(8) = 3}$ and finally $\mathbf{\log_2 8 = 3}$.
- These can all be read as **"The power/exponent of 2 that equals 8 is 3"**.
- The abbreviation "log" is used for the word **'logarithm'**, which is a synonym for the word 'power'.
- **So . . . a logarithm is an exponent!**

Back to step 3 above . . . $x = 2^y$ can be converted to logarithmic form $y = \log_2 x$. Therefore, if $f(x) = 2^x$, then $f^{-1}(x) = \log_2 x$. Notice then, that the inverse of an exponential function is a logarithmic function and vice-versa.

Exponential Equations vs. Logarithmic Equations



RESTRICTIONS: $b \neq 1$, $b > 0$, $n > 0$

EXAMPLES: Evaluate the expression:

A. $\log_2 32$ ⟶ 2 to what power gives you 32 ?

$32 = 2^p$

$\boxed{p = 5}$

B. $\log_3 81$ ⟶ 3 to what power gives you 81?

$81 = 3^p$

$\boxed{p = 4}$

PRACTICE: **Rewrite the equation in exponential form.**

1. $\log_2 8 = 3$ $2^3 = 8$

2. $\log_5 25 = 2$ $5^2 = 25$

3. $\log_{\frac{1}{2}} 16 = -4$ $\frac{1}{2}^{-4} = 16$

4. $\log_3 \frac{1}{27} = -3$ $3^{-3} = \frac{1}{27}$

PRACTICE: **Rewrite the equation in logarithmic form.**

5. $2^4 = 16$ $\log_2 16 = 4$

6. $4^6 = 4096$ $\log_4 4096 = 6$

7. $6^{-3} = \frac{1}{216}$ $\log_6 \frac{1}{216} = -3$

8. $25^{\frac{3}{2}} = 125$ $\log_{25} 125 = \frac{3}{2}$

$2^x=32$ $2^x=2^5$ $x=5$

$\log_b 1=0$ $\log_b b=1$

PRACTICE: Evaluate the expression without using a calculator.

9. $\log_2 32 = x$ 5

10. $\log_8 64$ 2

11. $\log_7 1$ 0

12. $\log_8 8$ 1

13. $\log_4 \frac{1}{16}$ −2

14. $\log_{16} 2 = x$ $16^x = 2$ $2^{4x}=2^1$ $4x=1$ $x=$ $\frac{1}{4}$

15. $\log_{\frac{1}{4}} \frac{1}{64}$ 3

16. $\log_{121} 11$ $\frac{1}{2}$ $121^x = 11$ $11^{2x}=11^1$ $2x=1$ $x=\frac{1}{2}$

Graphs of Logarithmic Functions

The graph of $y = \log_b(x-h) + k$ has the following characteristics:

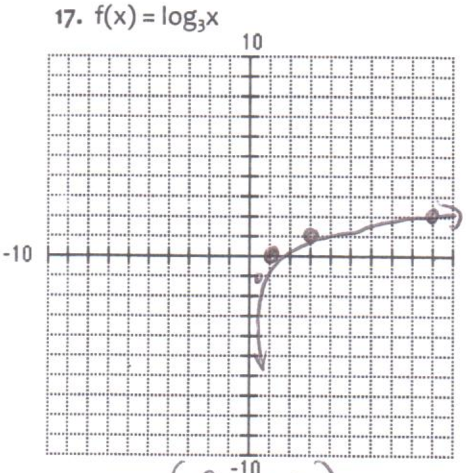
- The line $x = h$ is a vertical asymptote
- The domain is $x > h$ and the range is the set of all real numbers.
- If $b > 1$, the graph is the inverse of an exponential growth function.
- If $0 < b < 1$, the graph is the inverse of an exponential decay function.

Graph each function.

x	y	
$\frac{1}{5}$	$2\cdot -1 -1$	−3
1	$2\cdot 0 -1$	−1
5	$2\cdot 1 -1$	1

$\log_3 x$ → 3 & ↑2

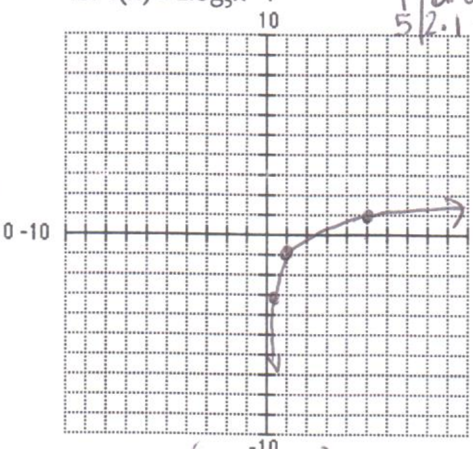
17. $f(x) = \log_3 x$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

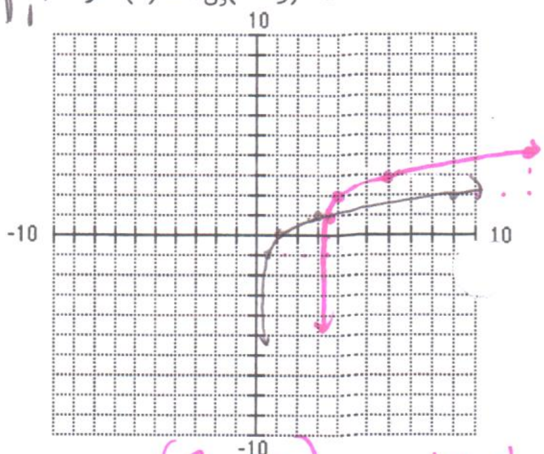
18. $f(x) = 2\log_5 x - 1$



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

19. $f(x) = \log_3(x-3) + 2$

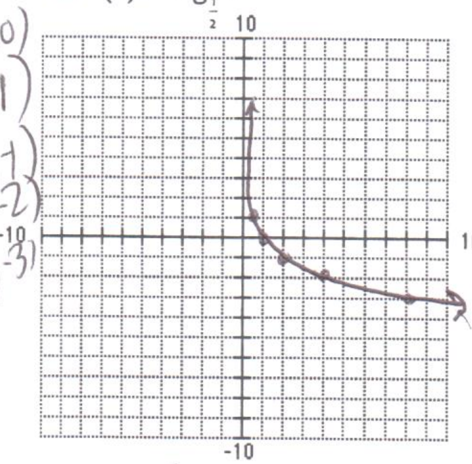


Domain: $(3, \infty)$ Asympt $x=3$

Range: $(-\infty, \infty)$

20. $f(x) = \log_{\frac{1}{2}} x$

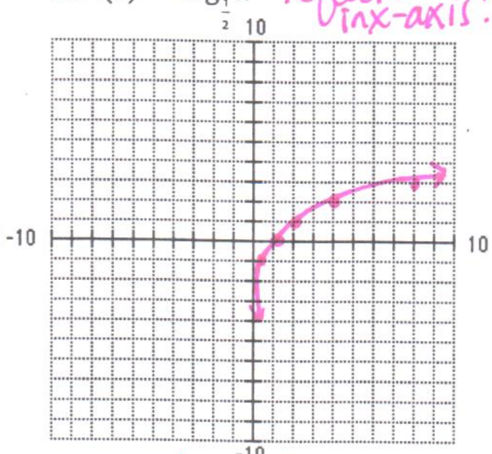
(1,0) $(\frac{1}{2},1)$ (2,−1) (4,−2) (8,−3)



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

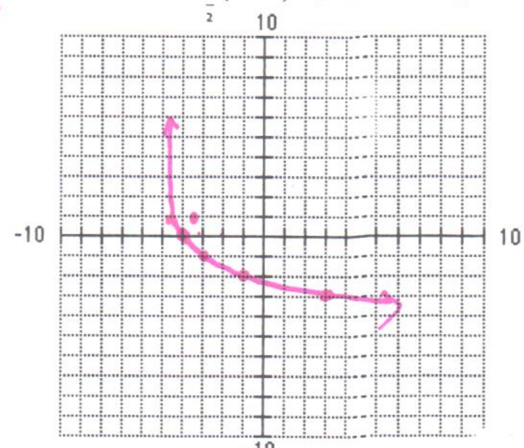
21. $f(x) = -\log_{\frac{1}{2}} x$ reflect #20 in x-axis!



Domain: $(0, \infty)$

Range: $(-\infty, \infty)$

22. $f(x) = \log_{\frac{1}{2}}(x+5)$ #20 ← 5



Domain: $(-5, \infty)$

Range: $(-\infty, \infty)$