

8-4 Trigonometry

Master E

- Trigonometry involves the ratio of sides of a **right triangle**!
- It is used to find the measures of the sides and/or angles of any right triangle when it's not a special triangle and you can't use the Pythagorean Theorem because you only have 1 side instead of 2 sides.
- Trigonometric ratios are related to the acute angles of a right triangle.
- The most common trigonometric ratios are:
sine (Sin), cosine (Cos), and tangent (Tan).

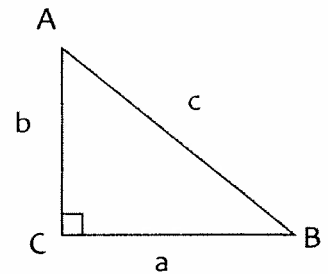
“Some Old Horse Came A Hoppin’ Through Our Alley” or “SOHCAHTOA”

Memorize these formulas!

EXAMPLES:

$$\begin{aligned} \sin \angle &= \frac{\text{opposite}}{\text{hypotenuse}} \left(\frac{O}{H} \right) \\ \cos \angle &= \frac{\text{adjacent}}{\text{hypotenuse}} \left(\frac{A}{H} \right) \\ \tan \angle &= \frac{\text{opposite}}{\text{adjacent}} \left(\frac{O}{A} \right) \end{aligned}$$

$$\begin{aligned} \sin A &= \frac{a}{c} & \sin B &= \frac{b}{c} \\ \cos A &= \frac{b}{c} & \cos B &= \frac{a}{c} \\ \tan A &= \frac{a}{b} & \tan B &= \frac{b}{a} \end{aligned}$$

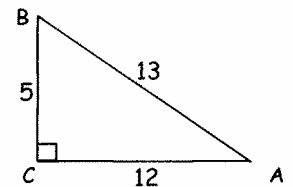


Example: Find $\sin A$, $\cos A$, and $\tan A$. Express each ratio as a decimal to the nearest thousandth.

$$\begin{aligned} \sin A &= \frac{\text{opposite leg}}{\text{hypotenuse}} \\ &= \frac{BC}{AB} \\ &= \frac{5}{13} \\ &\approx 0.385 \end{aligned}$$

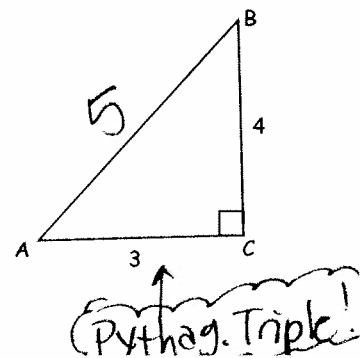
$$\begin{aligned} \cos A &= \frac{\text{adjacent leg}}{\text{hypotenuse}} \\ &= \frac{AC}{AB} \\ &= \frac{12}{13} \\ &\approx 0.923 \end{aligned}$$

$$\begin{aligned} \tan A &= \frac{\text{opposite leg}}{\text{adjacent leg}} \\ &= \frac{BC}{AC} \\ &= \frac{5}{12} \\ &\approx 0.417 \end{aligned}$$

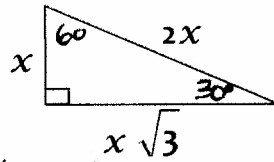
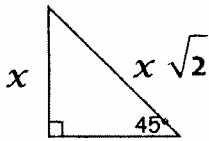


1-6: Find Sine, Cosine, & Tangent Ratios. Express each as a fraction and as a decimal to the nearest hundredth.

$$\begin{aligned} 1. \sin A &= \frac{4}{5} = 0.80 & 4. \sin B &= \frac{3}{5} = 0.60 \\ 2. \cos A &= \frac{3}{5} = 0.60 & 5. \cos B &= \frac{4}{5} = 0.80 \\ 3. \tan A &= \frac{4}{3} = 1.33 & 6. \tan B &= \frac{3}{4} = 0.75 \end{aligned}$$



7-15: Use the special right triangles to find trigonometric ratios in simplified radical form.



$$7. \sin 45^\circ = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$10. \sin 30^\circ = \frac{x}{2x} = \frac{1}{2}$$

$$13. \sin 60^\circ = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$$

$$8. \cos 45^\circ = \frac{x}{x\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$11. \cos 30^\circ = \frac{x\sqrt{3}}{2x} = \frac{\sqrt{3}}{2}$$

$$14. \cos 60^\circ = \frac{x}{2x} = \frac{1}{2}$$

$$9. \tan 45^\circ = \frac{x}{x} = 1$$

$$12. \tan 30^\circ = \frac{x}{x\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$15. \tan 60^\circ = \frac{x\sqrt{3}}{x} = \sqrt{3}$$

*What do you notice about the sine and cosine of complementary angles? they are the same!

Solve a Right Triangle: When you use the given measures in a right triangle to find the unknown sides and angle measures. You need to have 2 side lengths or one side length and the measure of one acute angle in order to solve the right triangle. All right triangles can be solved using trigonometry!

16-17: Use trigonometry to find each side length rounded to the nearest tenth. Put your calculator in degree mode!

16. $\sin 48 = \frac{x}{37} \Rightarrow x = 37 \cdot \sin 48$
 $\cos 48 = \frac{y}{37} \Rightarrow y = 37 \cdot \cos 48$
 $x \approx 27.5$ $y \approx 24.8$

17. $\tan 31 = \frac{x}{28} \Rightarrow x = 28 \cdot \tan 31$
 $\cos 31 = \frac{28}{y} \Rightarrow y = \frac{28}{\cos 31}$
 $x \approx 16.8$ $y \approx 32.7$

Inverse Trigonometric Ratios: When you know the sine, cosine, or tangent of an acute angle, you can use the calculator to find the measure of an angle, which is the inverse of the trigonometric ratio.

- **Inverse sine:** If $\sin A = x$, then $\sin^{-1}x = m\angle A$
- **Inverse Cosine:** If $\cos A = x$, then $\cos^{-1}x = m\angle A$
- **Inverse Tangent:** If $\tan A = x$, then $\tan^{-1}x = m\angle A$

18-19: Use inverse trigonometry to find each angle measure rounded to the nearest degree.

18. $\sin x = \frac{15}{y}$
 $\cos x = \frac{12}{y}$
 $\tan x = \frac{15}{12}$
 $\tan^{-1}\left(\frac{15}{12}\right) = x$
 $x \approx 51^\circ$ $y \approx 19.2$

19. $\tan y = \frac{12}{20} \Rightarrow \tan^{-1}\left(\frac{12}{20}\right) = y$
 $\tan x = \frac{20}{12} \Rightarrow \tan^{-1}\left(\frac{20}{12}\right) = x$
 $x \approx 59^\circ$ $y \approx 31^\circ$
 ✓ to make sure they are complementary $85!$

20: Do the real-life application.

20. A support wire is attached to the top of a 150 m radio tower. The wire is 190 m long. Find the measure to the nearest degree of the angle the wire makes with the ground.

$$\sin x = \frac{150}{190} \quad \sin^{-1}\left(\frac{150}{190}\right) = x$$

$$x \approx 52^\circ$$

