8-4 Solving Logarithmic Equations

Property of Equality: **If $\log_b m = \log_b n$, then $m = n$**
where b, m, & n > 0 and $b \neq 1$

Use this when you can set one logarithm equal to another logarithm and the bases are the same.

STEPS TO SOLVE USING THE PROPERTY OF EQUALITY:

1. Rewrite the equation so there is one logarithm on each side.
2. Set the "numbers" equal to each other.
3. Solve the "simple equation".
4. Check the solution by substituting the value into the ***original*** equation!

Solve each logarithmic equation. *Round to three decimal places when necessary.*

1. $\log_3(4x - 9) = \log_3(2x - 3)$

$4x-9=2x-3$
$2x=6$
$x=3$

2. $\log(x^2 - 1) = \log(x + 5)$

$x^2-1=x+5$
$x^2-x-6=0$
$(x-3)(x+2)$
$x=3, -2$

3. $\log_4(3x - 1) - \log_4(2x + 3) = 0$

$\log_4(3x-1)=\log_4(2x+3)$
$3x-1=2x+3$
$x=4$

4. $\log x = \log(5x - 10)$

$x=5x-10$
$-4x=-10$
$x=2.5$

5. $\log_2(x^2 - 6) = \log_2(2x + 2)$

$x^2-6=2x+2$
$x^2-2x-8=0$
$(x-4)(x+2)=0$
$x=4, -2$ ext.

6. $\log_4(x + 1) = \log_4(2x + 3)$

$x+1=2x+3$
$-2=x$ ∅ ext.

7. $\log_9(3 - x) = \log_9(5x - 15)$

$3-x=5x-15$
$18=6x$
$x=3$ ext. ∅

8. $\log_5(3x^2) = \log_5(2x + 1)$

$3x^2=2x+1$
$3x^2-2x-1=0$
$(3x+1)(x-1)=0$
$x=-\frac{1}{3}, 1$

Converting from Exponential to Logarithmic Form: If $n = b^p$, then $p = \log_b n$
where b & $n > 0$ and $b \neq 1$

Use this method when there is only ONE logarithm in the equation.

STEPS TO SOLVE BY CONVERTING THE FORM:

1. Isolate the logarithm.
2. Rewrite the equation in exponential form.
3. Solve the "simple equation".
4. Check the solution by substituting the value into the ***original*** equation!

Watch this video to help do these 4 problems: http://www.youtube.com/watch?v=NjF8XdaC9sM

1. $\log_4 64 = x$

$4^x = 64$

$4^x = 4^3$

$x = 3$

2. $\log_2\left(\frac{1}{16}\right) = x$

$2^x = \frac{1}{16}$

$2^x = 2^{-4}$

$x = -4$

3. $\log_5 x = 3$

$5^3 = x$

$x = 125$

4. $\log_x 81 = 4$

$x^4 = 81$

$x^4 = 3^4$

$x = 3$

Solve each logarithmic equation. *Round to three decimal places when necessary.*

5. $7\log x = 21$

$\log x = 3$

$10^3 = x$

$x = 1000$

6. $\log_2 x = 1.5$

$2^{1.5} = x$

$x = 2.828$

7. $2 + \log_2 3x = 8$

$\log_2 3x = 6$

$2^6 = 3x$

$x = \frac{64}{3} = 21.\overline{3}$

8. $\log(2x + 1) + 4 = 5$

$\log(2x+1) = 1$

$10 = 2x + 1$

$9 = 2x$

$x = 4.5$

9. $\log_3 2x = -2$

$3^{-2} = 2x$

$\frac{1}{9} = 2x$

$\frac{1}{18} = x = .05\overline{5}$

10. $\log_8 (x - 5) = \frac{2}{3}$

$8^{\frac{2}{3}} = x - 5$

$4 = x - 5$

$x = 9$

11. $\log_x (5x) = 2$

$x^2 = 5x$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0, 5$

ext. $x = 5$

base $\neq 0$!

12. $\log_{25}\left(\frac{x}{2}\right) = \frac{1}{2}$

$25^{\frac{1}{2}} = \frac{x}{2}$

$\pm 5 = \frac{x}{2}$

$x = 10, -10$ ext.

***13.** $\log_2 32 = 3x$

$2^{3x} = 32$

$2^{3x} = 2^5$

$3x = 5$

$x = \frac{5}{3} = 1.\overline{6}$