8-5 Properties of Logarithms

PRODUCT PROPERTY	QUOTIENT PROPERTY	POWER PROPERTY
Introduction: $\log_3(9 \bullet 27) = \log_3(3^2 \bullet 3^3) =$ $\log_3 3^{2+3} = 2 + 3 = 5$ When you multiply monomials with like bases, you **ADD** the exponents!!	**Introduction:** $\log_3\left(\frac{81}{27}\right) = \log_3\left(\frac{3^4}{3^3}\right) =$ $\log_3 3^{4-3} = 4 - 3 = 1$ When you divide monomials with like bases, you **SUBTRACT** the exponents!	**Introduction:** $\log_3 9^4 = \log_3(3^2)^4 =$ $\log_3 3^{2 \bullet 4} = 2 \bullet 4 = 8$ When you raise a monomial to a power, you **MULTIPLY** the exponents!
Conclusion: Since a logarithm is an exponent, then you "expand" and add the logs.	**Conclusion:** Since a logarithm is an exponent, then you "expand" and subtract the logs.	**Conclusion:** Since a logarithm is an exponent, then you multiply the power times the log.
PRODUCT Property $\log_b mn = \log_b m + \log_b n$ $m > 0, n > 0, \& \ b \neq 1$	**QUOTIENT Property** $\log_b \frac{m}{n} = \log_b m - \log_b n$ $m > 0, n > 0, \& \ b \neq 1$	**POWER Property** $\log_b m^p = p \log_b m$ p must be real, $m > 0, b > 0$ and $b \neq 1$
Example: $\log 5 + \log 3 = \log x$ $\log(5 \bullet 3) = \log x$ $\log 15 = \log x$ $15 = x \rightarrow \boxed{x = 15}$	**Example:** $\log_5 8 - \log_5 2 = \log_5 2x$ $\log_5\left(\frac{8}{2}\right) = \log_5 2x$ $\frac{8}{2} = 2x \rightarrow 4 = 2x \rightarrow \boxed{x = 2}$	**Example:** Evaluate $\log_3 9^4$ $\log_3 9^4 = 4\log_3 9$, since $\log_3 9 = 2$ (because $3^2 = 9$) $\log_3 9^4 = 4 \bullet 2 = \boxed{8}$

Use the properties of logs to rewrite the expression in terms of log 3 and log 4. Then use log 3 ≈ 0.477 and log 4 ≈ 0.602 to approximate the expression.

1. $\log\left(\frac{3}{4}\right)$ $\log 3 - \log 4$ $.477 - .602$ $-.125$	2. $\log 12$ $\log 3 \cdot 4$ $\log 3 + \log 4$ $.477 + .602$ 1.079	3. $\log 9$ $\log 3^2 = 2\log 3$ $2(.477)$ $.954$
4. $\log 16$ $\log 4^2 = 2\log 4$ $2(.602)$ 1.204	5. $\log\left(\frac{1}{4}\right)$ $\log 1 - \log 4$ $0 - .602$ $-.602$	6. $\log\left(\frac{4}{27}\right)$ $\log 4 - 3\log 3$ $.602 - 3(.477)$ $-.829$

Expand each expression.

7. $\log_6 3x$ $\log_6 3 + \log_6 x$	8. $\log_2 \frac{x}{5}$ $\log_2 x - \log_2 5$	9. $\log xy^2$ $\log x + 2\log y$
10. $\log_4 \frac{xy}{3}$ $\log_4 x + \log_4 y - \log_4 3$	11. $\log_3 \sqrt{x}yz$ $\frac{1}{2}\log_3 x + \log_3 y + \log_3 z$	12. $\log_5 2\sqrt{x}$ $\log_5 2 + \frac{1}{2}\log_5 x$

13. $\log \frac{x^2}{4}$ $2\log x - \log 4$	**14.** $\log \frac{10}{\sqrt{x}}$ $\log 10 - \frac{1}{2}\log x$ $1 - \frac{1}{2}\log x$	**15.** $\log_2 \frac{x^2y}{z}$ $2\log_2 x + \log_2 y - \log_2 z$

Condense each expression.

16. $\log_3 7 - \log_3 x$ $\log_3 \frac{7}{x}$	**17.** $2\log_5 x + \log_5 3$ $\log_5 3x^2$	**18.** $\log_4 5 + \log_4 x + \log_4 y$ $\log_4 5xy$
19. $\frac{1}{2}\log x - \log 4$ $\log \frac{\sqrt{x}}{4}$	**20.** $\frac{2}{3}\log_2 x - 3\log_2 y$ $x^{(2/3)}$ $\log_2 \frac{\sqrt[3]{x^2}}{y^3}$	**21.** $\log_3 4 + 2\log_3 x - \log_3 5$ $\log_3 \frac{4x^2}{5}$

The pH of a patient's blood can be calculated using the Henderson-Hasselbach Formula, $pH = 6.1 + \log \frac{B}{C}$, where B is the concentration of bicarbonate and C is the concentration of carbonic acid. The normal pH blood is approximately 7.4.

22. Expand the right side of the formula. $pH = 6.1 + \log B - \log C$	**23.** A patient has a bicarbonate concentration of 24 and a carbonic acid concentration of 1.9. Find the pH of the patient's blood. $6.1 + \log 24 - \log 1.9$ 7.2	**24.** Is the patient's pH above or below normal? below normal

Solve each equation. Round to three decimal places when necessary.

25. $\log_2 x + \log_2(x+1) = 1$ $\log_2 x(x+1) = 1$ $2 = x^2 + x$ $0 = x^2 + x - 2$ $(x+2)(x-1)$ $x = -2, 1$ $x = 1$	**26.** $\log(x+1) - 3 = \log x$ $\log(x+1) - \log x = 3$ $\log \frac{x+1}{x} = 3$ $1000 = \frac{x+1}{x}$ $1000x = x + 1$ $999x = 1$ $\frac{1}{999}$ or $.001$
27. $\log(x+2) + \log(x-3) = \log(x+29)$ $\log(x+2)(x-3) = \log(x+29)$ $x^2 - x - 6 = x + 29$ $x^2 - 2x - 35 = 0$ $(x-7)(x+5)$ $x = 7, -5$	**28.** $\log_8(t+10) - \log_8(t-1) = \log_8 12$ $\log_8 \frac{t+10}{t-1} = \log_8 12$ $\frac{t+10}{t-1} = 12 \Rightarrow 12t - 12 = t + 10$ $11t = 22$ $t = 2$
29. $3\log_5(x^2+9) - 6 = 0$ $3\log_5(x^2+9) = 6$ $\log_5(x^2+9) = 2$ $25 = x^2 + 9$ $16 = x^2$ $x = \pm 4$	**30.** $\log_2 x = 5\log_2 2 - \log_2 8$ $\log_2 x = \log_2 \frac{2^5}{8}$ $x = 4$ $\log_2 x = 5 - 3$ $\log_2 x = 2$ $4 = x$