8-6 & 8-7 Common and Natural Logarithms

Common Logarithm: a log with base 10 $\log_{10} x$ is written as $\log x$	**Natural Logarithm:** a log with base e $\log_e x$ is written as $\ln x$

$\boxed{e}$ is $\boxed{2nd}\boxed{\div}$! (Calc.)

Evaluate the expression without using a calculator:

1. $\log 10{,}000$ $10^x = 10{,}000$ ~~3~~ 4	2. $\log \frac{1}{10}$ -1	3. $\log 1$ 0	4. $\ln e^4$ 4	5. $\ln \frac{1}{e^2}$ e^{-2}	6. $\ln e^{x+3}$ $x+3$

Use the calculator to evaluate each expression. Round your result to 3 decimal places.

7. $\log 6$ .778	8. $\log (0.4)$ $-.398$	9. $\ln 8$ 2.079	10. $\ln 6.12$ 1.812
11. e^5 $\boxed{2nd}\boxed{ln}$ 148.413	12. $e^{-\frac{1}{3}}$ .717	13. $e^{-1.4}$ .247	14. $e^{\sqrt{2}}$ 4.113

State whether the function is an example of exponential growth or exponential decay.

15. $f(x) = 2e^{3x}$ Growth	16. $f(x) = e^{-3x} = \frac{1}{e^{3x}} = \left(\frac{1}{e^3}\right)^x$ Decay	17. $f(x) = 2e^{-3x}$ $2 \cdot \left(\frac{1}{e^3}\right)^x$ Decay
18. $f(x) = \frac{1}{5}e^{5x}$ Growth	19. $f(x) = \frac{1}{2}e^{-x}$ $\frac{1}{2}\left(\frac{1}{e}\right)^x$ Decay	20. $f(x) = 4e^{5x}$ Growth

* Remember ... e is a # ≈

HOW TO SOLVE AN EQUATION USING COMMON AND/OR NATURAL LOGS AND THE PROPERTIES:

IF YOU CAN'T GET LIKE BASES on both sides of equation:

1) Undo any operations to get the value with the variable exponent alone on one side.
2) Then, take the common log or the natural log of both sides of the equation in order to remove the variable from the exponent's position OR you can put the expression into logarithmic form and then use the change of base formula to finish the problem!

Change of Base Formula: $\log_a n = \frac{\log_b n}{\log_b a}$ or $\frac{\log n}{\log a}$ or $\frac{\ln n}{\ln a}$

3) Solve for the variable by using the properties of logarithms. ***(YOU HAVE TO MEMORIZE THESE!)***
4) If you need to use a calculator for the final step, write the final expression solved for the variable that you will plug into the calculator. *Round only at the end so your answer will not be off!*
5) Check the solution (*the base and the number can't be negative, only the exponent/answer!*)

An Example from the book on page 517

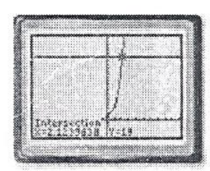
Example 3 Solve Exponential Equations Using Logarithms

Solve $4^x = 19$. Round to the nearest ten-thousandth.

$4^x = 19$	Original equation
$\log 4^x = \log 19$	Property of Equality for Logarithmic Functions
$x \log 4 = \log 19$	Power Property of Logarithms
$x = \frac{\log 19}{\log 4}$	Divide each side by log 4.
$x \approx 2.1240$	Use a calculator.

The solution is approximately 2.1240.

CHECK You can check this answer graphically by using a graphing calculator. Graph the line $y = 4^x$ and the line $y = 19$. Then use the **CALC** menu to find the intersection of the two graphs. The intersection is very close to the answer that was obtained algebraically. ✓



$[-10, 10]$ scl: 1 by $[-5, 25]$ scl: 1

The graphing method above is a neat way to check!

or just put it in logarithmic form: $\ln_e 9 = x$ $e^x = 9$!

& type in ln 9

* Do not round on each step! Only round on the last answer!

Solve each equation. Round to three decimal places when necessary.

21. $e^x = 9$ $\ln e^x = \ln 9$ $x \ln e = \ln 9$ $x = \frac{\ln 9}{\ln e}$ $x = 2.197$	**22.** $2^x + 7 = 10$ $2^x = 3$ $\log_2 3 = x$ change of base: $\frac{\log 3}{\log 2}$ $x = 1.585$	**23.** $3^{2x-5} = 7$ $\log_3 3^{2x-5} = \log_3 7$ $2x-5 = \frac{\log 7}{\log 3}$ $2x-5 = 1.771\ldots$ $2x = 6.771\ldots$ $x = 3.386$
24. $\frac{2}{3}e^{3x} + 1 = 10$ $\frac{2}{3}e^{3x} = 9$ $e^{3x} = 9 \cdot \frac{3}{2}$ $e^{3x} = 13.5$ $\ln 13.5 = 3x$ $2.603\ldots = 3x$ $x = .868$	**25.** $2^{x+1} = 3^{2x}$ $\log_2 2^{x+1} = \log_2 3^{2x}$ $x+1 = 2x\left(\frac{\log 3}{\log 2}\right)$ $x+1 = 3.1699x$ $1 = 2.1699x$ $x = .461$	**26.** $e^{x-3} = 10^{4-x}$ * $\ln e^{x-3} = \ln 10^{4-x}$ $x-3 = (4-x)(\ln 10)$ $x-3 = 4(\ln 10) - x(\ln 10)$ $x-3 = 9.210 - 2.303x$ $3.303x = 12.210$ $x = 3.697$
27. $\ln x = 5$ $e^5 = x$ $x = 148.413$	**28.** $7\log x = 21$ $\log x = 3$ $10^3 = x$ $x = 1000$	**29.** $4 - \ln x = 1$ $-\ln x = -3$ $\ln x = 3$ $e^3 = x$ $x = 20.086$
30. $2 + \log_2 3x = 8$ $\log_2 3x = 6$ $2^6 = 3x$ $64 = 3x$ $x = 21.333$	**31.** $\log(2x+1) + 4 = 5$ $\log(2x+1) = 1$ $10 = 2x+1$ $9 = 2x$ $x = 4.5$	**32.** $\ln 4x - 6 = 8$ $\ln 4x = 14$ $e^{14} = 4x$ $\frac{e^{14}}{4} = x$ $x = 300651.071$
33. $\log(x+2) + \log(x-3) = \log(x+29)$ $\log(x+2)(x-3) = \log(x+29)$ $x^2 - x - 6 = x + 29$ $x^2 - 2x - 35 = 0$ $(x-7)(x+5) = 0$ $x = 7, -5 \rightarrow$ extr. $x = 7$	**34.** $\log_2 x + \log_2(x+1) = 1$ $\log_2 x(x+1) = 1$ $2 = x^2 + x$ $x^2 + x - 2 = 0$ $(x+2)(x-1) = 0$ $x = -2, 1$ $x = 1$	**35.** 23. $\log_2(x+1) = \log_4(2x+3)$ $\log_2(x+1) = \frac{\log_2(2x+3)}{\log_2 4}$ $\log_2(x+1) = \frac{\log_2(2x+3)}{2}$ $2\log_2(x+1) = \log_2(2x+3)$ $\log_2(x+1)^2 = \log_2(2x+3)$

change the base!

$(x+1)^2 = (2x+3)$

$\leftarrow x^2 + 2x + 1 = 2x + 3$

$x = 1.414 \leftarrow x^2 = 2$, $x = \pm\sqrt{2}$