8-8 Applications of Exponential & Logarithmic Functions

Name Master Ep

Date ____________ Block ______

1. Determine the balance of a retirement account after 20 years if $5,000 was invested at 6.05% interest compounded weekly.

52 in a yr $A=P(1+\frac{r}{n})^{nt}$ $A=5000(1+\frac{.0605}{52})^{(52\cdot 20)}=\$16,755.63$

2. When Angelina was born, her grandparents deposited $3000 into a college savings account paying 4% interest compounded continuously. Angelina never made any deposits or withdrawals from the account.

a. What will the balance be after 10 years?

$A=Pe^{rt}$ $A=3000e^{(.04\cdot 10)}$ $4,475.47

b. If her grandparents want Angelina to have $10,000 after 18 years, how much would they need to invest?

$10{,}000=Pe^{(04\cdot 18)}$ $4867.52

3. The first U.S. Census was conducted in 1790. At that time, the population was 3,929,214. Since then, the U.S. population has grown by approximately 2.03% annually.

*2010 Census: 281,421,906 (9.7% increase from 2000 census)

a. Write a model which represents population growth where y represents population and x represents time in years.

$y=3{,}929{,}214(1+.0203)^x$ $y=3{,}929{,}214(1.0203)^x$

b. What is the "growth factor?" 1.0203

c. Graph the model using the calculator. Where is the y-intercept?

(0, 3,929,214)

d. What does this point represent in the context of population? the initial population in 1790

e. Assuming population continues to grow at this rate, estimate the population in 2015.

$y=3{,}929{,}214(1.0203)^{225}$

$\begin{array}{r} 2015 \\ -1790 \\ \hline 225 \end{array}$

361,476,592.4

★**4.** A sequence of numbers follows a pattern in which the next number is 115% of the previous number. The first number in the pattern is 12. 12, 13.8, 15.87, 18.2505, 42.21

a. Write the function which represents the sequence. $y=12(1.15)^{x-1}$

b. What is the value of the tenth number? Round to the nearest hundredth.

42.21 10th term | 48.55 ← $y=12(1.15)^x$ 11th term

$4.458 = 2.556(1+r)^{30}$ $(1.744)^{\frac{1}{30}} = ((1+r)^{30})^{\frac{1}{30}}$ $1.018... = 1+r$

5. In 1950, the world population was about 2.556 billion. By 1980, it had increased to about 4.458 billion.

a. Write an exponential function to model the world population, y, in billions for 1950 to 1980.

$\boxed{y = 2.556(1.019)^x}$

b. Suppose the population continued to grow at that rate. Estimate the population in 2000.

$y = 2.556(1.019)^{50}$ $\boxed{6.55 \text{ billion}}$ $2000 - 1950 = 50$

c. In 2000, the world population was about 6.08 billion. Compare your estimate to the actual population.

I was off by .47 billion.

d. Use the equation you wrote in Part a above to estimate the world population in the year 2020.

$y = 2.556(1.019)^{70}$ $\boxed{9.54 \text{ billion}}$ $2020 - 1950 = 70 = x$

e. How accurate to you believe this estimate to be? Explain your reasoning.

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6. The Richter scale measures earthquake intensity. The increase in intensity between each number is 10 times. For example, an earthquake with a rating of 7 is 10 times more intense than one measuring 6. The magnitude, M, of an earthquake is given by M = log x, where x represents the amplitude of the seismic wave causing ground motion.

$M = \log x$ (M: magnitude, x: amplitude)

a. How many times as great is the amplitude caused by an earthquake with a Richter scale rating of 8 as an aftershock with a Richter scale rating of 5?

$8 = \log x$ $5 = \log x$ $\frac{10^8}{10^5} = 10^3$ about 1000x as great as the aftershock.

tmp → $x = 10^8$ $x = 10^5$

b. In 1906, San Francisco was almost completely destroyed by a 7.8 magnitude earthquake. In 1911, an earthquake estimated at magnitude 8.1 occurred along the New Madrid fault in the Mississippi River Valley. How many times greater was the New Madrid earthquake than the San Francisco earthquake?

$\frac{10^{8.1}}{10^{7.8}} = 10^{0.3} = 2$ ≈ 2x as great as the San Francisco earthquake

7. The pH of a substance is defined as the concentration of hydrogen ions $[H^+]$ in moles. It is given by the formula $pH = \log \frac{1}{H^+}$. Find the amount of hydrogen in a liter of acid rain that has a pH of 4.2.

$PH = \log \frac{1}{H}$

$4.2 = \log \frac{1}{H}$

$4.2 = \log 1 - \log H$

$-4.2 = \log H$

$10^{-4.2} = H$

$\boxed{H = 0.0000631}$ mole of hydrogen in a liter of rain

8. For a certain strain of bacteria, the rate of continuous growth, k, is 0.728 when t is measured in days. Using the formulas $y = ae^{kt}$, how long will it take 10 bacteria to increase to 675 bacteria?

$y = ae^{kt}$

$675 = 10e^{0.728t}$

$67.5 = e^{0.728t}$

$\ln 67.5 = 0.728t$

$t = \frac{\ln 67.5}{0.728} = \boxed{5.8 \text{ days}}$