9-4 Rational Functions Review

Name Master
Date _____ Block ____

VERTICAL ASYMPTOTES

- When in the form $f(x) = \frac{a}{x-h} + k$, it is $x = h$
- If $f(x) = \frac{a(x)}{b(x)}$, there is a vertical asymptote whenever $b(x) = 0$

  Set the factors of the denominator equal to zero and solve for x.

HORIZONTAL ASYMPTOTES

- f(x) has as most ONE horizontal asymptote.
- When in the form $f(x) = \frac{a}{x-h} + k$, it is $y = k$
- When in the form $f(x) = \frac{ax^n + b}{cx^n + d}$ (degree of numerator **equals** degree of denominator), it is $y = \frac{a}{c}$
- When the degree of the numerator is **less than** the degree of the denominator, it is $y = 0$.
- When the degree of the numerator is **greater than** the degree of the denominator, then a *SLANT* asymptote exits: y = the denominator divided by the numerator minus the remainder.

ZEROS OF THE FUNCTION

- A zero of a rational function $f(x) = \frac{a(x)}{b(x)}$ occurs at every value of x for which $a(x) = 0$.

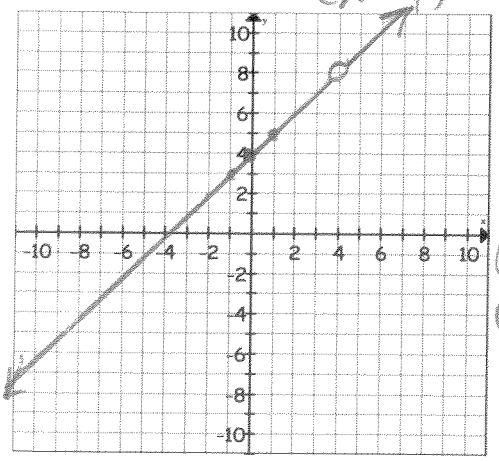
  Set the factors of the numerator equal to zero and solve to find out where the graph crosses the x-axis.

Practice Finding the Asymptotes, Zeros, & y-intercepts without a calculator:

Equation of the Function: Factor it first!	Vertical Asymptote(s):	Horizontal Asymptote(s):	Zeros of the Function:	y-intercept
1. $f(m) = \frac{x+2}{x-4}$	$x=4$	$y=1$	$(-2,0)$	$(0,-\frac{1}{2})$
2. $f(a) = \frac{2x^2}{x^2-9}$ $\frac{2x^2}{(x+3)(x-3)}$	$x=\pm 3$	$y=2$	$(0,0)$	$(0,0)$
3. $f(t) = \frac{-2}{x^2+2x+1}$ $\frac{-2}{(x+1)(x+2)}$	$x=-1, x=-2$	$y=0$	None	$(0,-2)$
4. $f(h) = \frac{-3x}{x-2}$	$x=2$	$y=-3$	$(0,0)$	$(0,0)$
5. $f(i) = \frac{x-2}{x}$	$x=0$	$y=1$	$(2,0)$	None
6. $f(s) = \frac{x-1}{x^2-8x+16}$ $\frac{x-1}{(x-4)^2}$	$x=4$	$y=0$	$(1,0)$	$(0,-\frac{1}{16})$
7. $f(c) = \frac{3}{x^2-1}$ $\frac{3}{(x+1)(x-1)}$	$x=\pm 1$	$y=0$	None	$(0,-3)$
8. $f(o) = \frac{x^2-x-6}{x^2-x-12}$ $\frac{(x-3)(x+2)}{(x-4)(x+3)}$	$x=4, x=-3$	$y=1$	$(3,0)(-2,0)$	$(0,\frac{1}{2})$
9. $f(o) = \frac{x+8}{x^2-36}$ $\frac{(x+8)}{(x+6)(x-6)}$	$x=\pm 6$	$y=0$	$(-8,0)$	$(0,-\frac{2}{9})$
10. $f(l) = \frac{-3x-6}{2x-12}$ $\frac{-3(x+2)}{2(x-6)}$	$x=6$	$y=-\frac{3}{2}$	$(-2,0)$	$(0,\frac{1}{2})$

Examples of Holes:

11. $f(x)=\dfrac{x^2-16}{x-4}$ $\dfrac{(x+4)(x-4)}{(x-4)}=x+4$

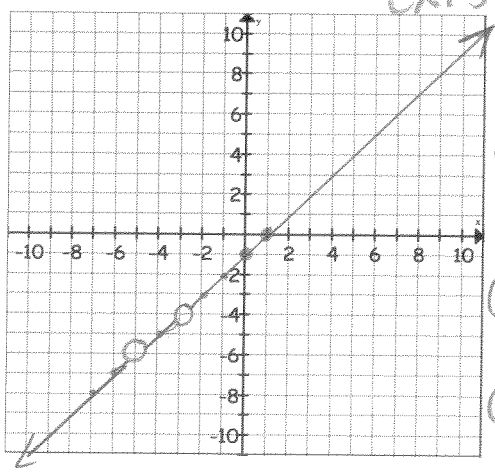


(4,8)

$(-\infty,4)\cup(4,\infty)$

$(-\infty,8)\cup(8,\infty)$

12. $f(x)=\dfrac{(x+5)(x^2+2x-3)}{x^2+8x+15}$ $\dfrac{(x+5)(x+3)(x-1)}{(x+5)(x+3)}$ $=x-1$



$(-5,-6)$

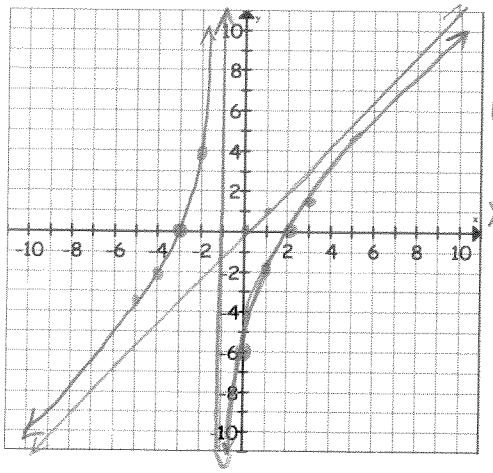
$(-3,-4)$

$(-\infty,-5)\cup(-5,$
$\cup(-3,\infty)$

$(-\infty,-6)\cup(-6,$
$\cup(-4,\infty)$

Examples of Slant Asymptotes:

13. $f(x)=\dfrac{x^2+x-6}{x+1}$ $(x+3)(x-2)$

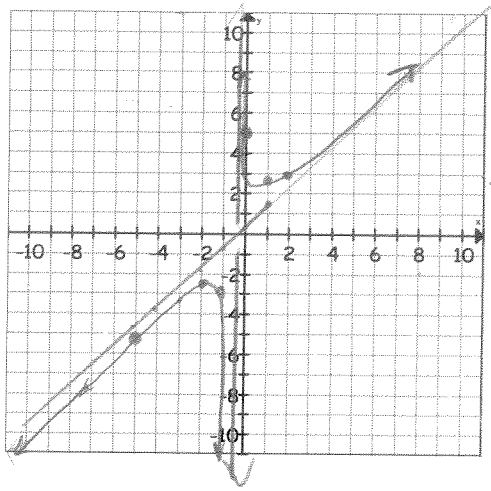


$x=-1$

$y=x$

$$x+1\overline{)\,x^2+x-6}$$
$$\underline{x^2+x}$$
$$0x-6$$

14. $f(x)=\dfrac{3x^2+2x+5}{3x+1}$



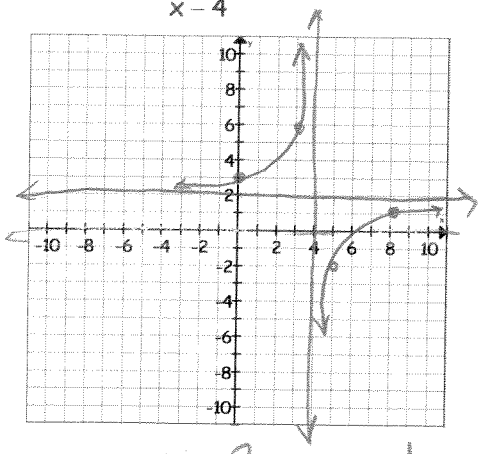
$y=x+\frac{1}{3}$

$x=-\frac{1}{3}$

$$3x+1\overline{)\,3x^2+2x+5}$$
$$\underline{3x^2+x}$$
$$x+5$$
$$\underline{x+\frac{1}{3}}$$

Review of other graphs:

15. $f(x)=\dfrac{-4}{x-4}+2$

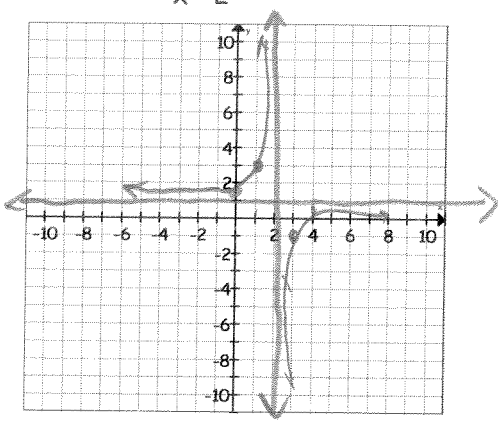


asymp: H: $y=2$ V: $x=4$

domain: $(-\infty,4)\cup(4,\infty)$

range: $(-\infty,2)\cup(2,\infty)$

16. $f(x)=\dfrac{x-3}{x-2}$

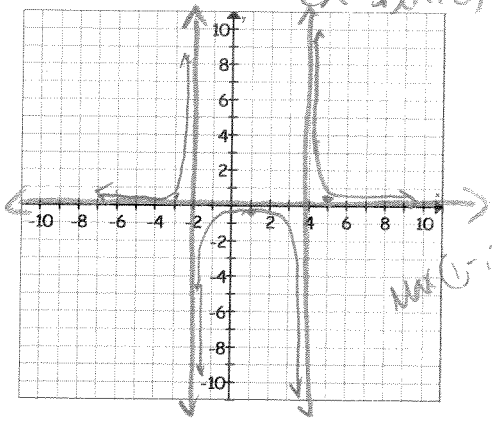


asymp: H: $y=1$ V: $x=2$

domain: $(-\infty,2)\cup(2,\infty)$

range: $(-\infty,1)\cup(1,\infty)$

17. $f(x)=\dfrac{3}{x^2-2x-8}$ $\dfrac{3}{(x-4)(x+2)}$



Max (-1, -3)

asymp: H: $y=0$ V: $x=4, x=-2$

domain: $(-\infty,-2)\cup(-2,4)\cup(4,\infty)$

range: $(-\infty,-3]\cup(0,\infty)$