Direct, Inverse, and Joint Variation

Name _Master E_
Date _____ Block _____

DIRECT VARIATION	INVERSE VARIATION	JOINT VARIATION
k is the **constant of variation** – it is the part that never changes!		
$y = kx,\ k \neq 0 \quad k = \dfrac{y}{x}$	$y = \dfrac{k}{x},\ k \neq 0 \quad k = xy$	$z = kxy,\ k \neq 0$
"y varies directly with x" as x increases, y increases Example: $C = \Pi d$ (Π is k)	" y varies inversely with x" as x increases, y decreases Example: $t = \dfrac{D}{r}$ (d is k)	"z varies jointly with x and y" as x and y increase, z increases as x and y decrease, z decreases Example: $A = \dfrac{1}{2} b \bullet h$ ($\dfrac{1}{2}$ is k)

Determining Variation Given an Equation:

Solve the given equation for one of the variables (usually y) and compare it the general forms above.

Examples:

    a. $xy = 10 \longrightarrow y = \dfrac{10}{x} \longrightarrow$ inverse variation

    b. $\dfrac{y}{3} = x \longrightarrow y = 3x \longrightarrow$ direct variation

    c. $x + y = 5 \longrightarrow y = -x + 5 \longrightarrow$ neither (Direct Variation always has a y-int. of 0)

Practice: Tell whether x and y show direct or inverse variation. *Remember to solve for y first!*

1. $x = \dfrac{y}{9}$ $y = 9x$ *Direct*	2. $y = \dfrac{1}{2}x$ *Direct*	3. $xy = 0.1$ $y = \dfrac{0.1}{x}$ *Inverse*	4. $y = x + 5$ *Neither*
5. $x = 4y$ $y = \dfrac{1}{4}x$ *Direct*	6. $x = \dfrac{5}{y}$ $y = \dfrac{5}{x}$ *Inverse*	7. $x = \dfrac{y}{3}$ $y = 3x$ *Direct*	8. $\dfrac{2}{x} = \dfrac{7}{y}$ $2y = 7x$ $y = \dfrac{7}{2}x$ *Direct*

Steps to Writing a Variation Equation:

1. Write the correct variation formula.
2. Substitute the given values into your equation and find k.
3. Rewrite the equation with replacing k with the value you found in #2.
4. Find the value of the variable by plugging in the new information into your variation equation.

Direct Variation: If y varies directly as x and y = -4 when $x = \dfrac{1}{3}$, then find y when x = 8.

$$y = kx \longrightarrow -4 = k\left(\dfrac{1}{3}\right) \longrightarrow k = -12 \longrightarrow \boxed{y = -12x} \longrightarrow y = -12(8) \longrightarrow \boxed{y = -96}$$

Inverse Variation: If y varies inversely as x and y = -2 when x = 5, then find y when x = 2.

$$y = \dfrac{k}{x} \longrightarrow -2 = \dfrac{k}{5} \longrightarrow k = -10 \longrightarrow \boxed{y = \dfrac{-10}{x}} \longrightarrow y = \dfrac{-10}{2} \longrightarrow \boxed{y = -5}$$

Joint Variation: If z varies jointly as x and y and z = 6 when x = 3 and y = 4, find z when x = 5 and y = 2.

$$z = kxy \longrightarrow 6 = k(3)(4) \longrightarrow 6 = 12k \longrightarrow k = \dfrac{1}{2} \longrightarrow \boxed{z = \dfrac{1}{2}xy} \longrightarrow z = \dfrac{1}{2}(5)(2) \longrightarrow z = \dfrac{1}{2}(10) \longrightarrow \boxed{z = 5}$$

Practice Problems

Find the value of k for each scenario. Then write an equation replacing k with the value you found. Finally, find the value of the variable being asked for.

9. If y varies directly as x and y = 8 when x = 2, find y when x = 6.

$y = kx$ $8 = k(2)$ $k = 4$ $\boxed{y = 4x}$ $y = 4(6)$ $\boxed{y = 24}$

10. If y varies directly as x and y = -16 when x = 6, find x when y = -4.

$y = kx$ $-16 = k(6)$ $\boxed{y = \frac{-8}{3}x}$ $-4 = \frac{-8}{3}x$ $x = -4\left(\frac{-3}{8}\right)$

$k = \frac{-16}{6} = \frac{-8}{3}$ $\boxed{x = \frac{3}{2}}$

11. If y varies directly as x and y = 132 when x = 11, find y when x = 33.

$y = kx$ $132 = k(11)$ $\boxed{y = 12x}$ $y = 12(33)$ $\boxed{y = 396}$

$k = 12$

12. If y varies jointly as x and z and y = 24 when x = 2 and z = 1, find y when x = 12 and z = 2.
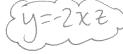
$y = kxz$ $24 = k(2)(1)$ $\boxed{y = 12xz}$ $y = 12(12)(2)$ $\boxed{y = 288}$

$k = 12$

13. If y varies jointly as x and z and y = 60 when x = 3 and z = 4, find y when x = 6 and z = 8.

$y = kxz$ $60 = k(3)(4)$ $\boxed{y = 5xz}$ $y = 5(6)(8)$ $\boxed{y = 240}$

$= k = 5$

14. If y varies jointly as x and z and y = 12 when x = -2 and z = 3, find y when x = 4 and z = -1.

$y = kxz$ $12 = k(-2)(3)$ $\boxed{y = -2xz}$ $y = -2(4)(-1)$ $\boxed{y = 8}$

$k = -2$

15. If y varies inversely as x and y = 16 when x = 4, find y when x = 3.

$y = \frac{k}{x}$ $16 = \frac{k}{4}$ $\boxed{y = \frac{64}{x}}$ $\boxed{y = \frac{64}{3}}$

$k = 64$

16. If y varies inversely as x and y = 3 when x = 5, find x when y = 2.5.

$y = \frac{k}{x}$ $3 = \frac{k}{5}$ $\boxed{y = \frac{15}{x}}$ $2.5 = \frac{15}{x}$ $2.5x = 15$ $\boxed{x = 6}$

$k = 15$

17. If y varies directly as z and inversely as x and y = 10 and z = 5 when x = 12.5, find z when y = 37.5 and x = 2.

$y = \frac{kz}{x}$ $10 = \frac{k(5)}{12.5}$ $\boxed{y = \frac{25z}{x}}$ $37.5 = \frac{25z}{2}$ $\boxed{z = 3}$

$125 = 5k$ $75 = 25z$

$k = 25$

Determining Variation Given a Table of Values:

- Multiply each of the pairs. If the product is the same, then the values show an **inverse variation.**
- Divide each of the pairs. If the quotient is the same, then the values show a **direct variation.**
- If neither the product nor quotient is the same, then there is **neither** direct nor inverse variation.

Example:

x	y
1.5	20
2.5	12
4	7.5
5	6

When multiplying, the products all equal 30. ⟶ $xy = 30$

Therefore, the values show an inverse variation.

*30 is the constant of variation (k) and the equation is $y = \dfrac{30}{x}$.

Practice: Tell whether x and y show direct variation, inverse variation, or neither. If is direct or inverse, find k and then write the equation of the function.

18.

x	y
5	15
8	24
1.5	4.5
0.5	1.5

$\dfrac{y}{x} = 3$

$y = 3x$

Direct

19.

x	y
3	5
5	21
4.5	16.25
7	45

Neither

20.

x	y
1	4
2	2
0.5	8
0.25	16

$xy = 4$

$y = \dfrac{4}{x}$

Inverse

21.

x	y
3	6
7	10
2.5	5.5
5.7	8.7

Neither

How to Solve Word Problems:

1. Read the problem and determine what type of variation is represented.
2. Write the variation formula and substitute the given values into your equation to find k.
3. Do not round k! If it is not a terminating decimal, write it as an improper fraction (*premature rounding can completely change the final answer and make it way off!*)
4. Rewrite the equation replacing k with the value you found in #2.
5. Find the value of the variable by plugging in the new information into your variation equation.
6. Write your final answer using the correct units.
7. If your answer is an amount of money, remember to round to the nearest hundredth.

Example:

The length S that a spring will stretch varies directly with the weight F that is attached to the string. If a spring stretches 20 inches with 25 pounds attached, how far will it stretch with 15 pounds attached?

$S = kF$ ⟶ $20 = k(25)$ ⟶ $k = \dfrac{20}{25} = .8$ ⟶ $S = .8F$ ⟶ $S = .8(15) =$ 12 inches

Word Problem Practice

22. The simple interest I (in dollars) for a savings account is jointly proportional to the product of the time t (in years) and the principal P (in dollars). After nine months, the interest on a principal of $3500 is $91.88. What will the interest on the account be after 5 years? $\frac{9}{12}$ P   I

$I = ktP$

$I = .035tP$

$91.88 = k\left(\frac{9}{12}\right)(3500)$

$91.88 = 2625k$

$K = .03500...$ * store for x

$I = .035(5)(3500)$

$I = \$612.53$

500 if you didn't store for x & used the rounded answer.

23. Boyle's Law states that for a constant temperature, the pressure P of a gas varies inversely with its volume V. A sample of hydrogen gas has a volume of 8.56 cubic liters at a pressure of 1.5 atmospheres. Find the volume of the hydrogen gas if the pressure changes to 1.2 atmospheres.

$P = \frac{k}{V}$

$P = \frac{12.84}{V}$

$1.2 = \frac{12.84}{V}$

$1.5 = \frac{k}{8.56}$

$1.2V = 12.84$

$K = 12.84$

$V = 10.7$ cubic liters

24. The illumination in luxes (I) of a surface varies inversely with the square of the distance in meters (d) from the light source to the surface. One meter away from a certain light the illumination is 750 luxes. Write an equation that relates I and d. Then find the illumination at a distance of 2 meters.

$I = \frac{k}{d^2}$

$I = \frac{750}{d^2}$

$I = \frac{750}{2^2} = \frac{750}{4}$

$750 = \frac{k}{1}$

$K = 750$

$I = 187.5$ luxes

25. The power in watts of an electrical circuit varies jointly as the resistance and the square of the current. For a 600-watt microwave oven that draws a current of 5.0 amperes, the resistance is 24 ohms. What is the resistance of a 200-watt refrigerator that draws a current of 1.7 amperes?

$P = krc^2$

$P = rc^2$

$600 = k(24)(5)^2$

$600k = 600$

$k = 1$

$200 = r(1.7)^2$

$200 = 2.89r$

$r = 69.20$ ohms