11-2 Arithmetic Series

Name______________________
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ARITHMETIC SERIES: the sum of the terms of an arithmetic sequence.

SUM OF AN ARITHMETIC SEQUENCE: $a_1 + a_2 + a_3 + \ldots + a_n \longrightarrow \boxed{S_n = \frac{n}{2}(a_1 + a_n)}$

Notation: S_n = the Sum of n terms; n = the # of terms; a_1 = the 1st term; a_n = the nth (last) term

Remember: To find any term, you would use the nth term formula: $\boxed{a_n = a_1 + (n - 1)d}$
So if you missing a_n, another formula can be derived:

$$S_n = \frac{n}{2}(a_1 + a_1 + (n-1)d) \longrightarrow \boxed{S_n = \frac{n}{2}(2a_1 + (n-1)d)}$$

Examples done by me:

A. Find S_n for the sequence with $a_1 = 2$, $n = 12$, $a_n = 25$.
$S_{12} = \frac{12}{2}(2 + 25)$
$\boxed{S_{12} = 162}$

B. Find S_n for the sequence with $a_1 = 10$, $n = 25$, $a_n = 1000$.
$S_{25} = \frac{25}{2}(10 + 1000)$
$\boxed{S_{25} = 12,625}$

C. Find S_n given $a_1 = 4$, $d = 8$, $n = 10$.
$a_{10} = 4 + (10 - 1)8$
$a_{10} = 76$
$S_{10} = \frac{10}{2}(4 + 76)$
$\boxed{S_{10} = 400}$

Examples done by you: Find S_n for each arithmetic series described.

1. $a_1 = 60$, $a_n = -136$, & $n = 50$

$S_{50} = \frac{50}{2}(60 + -136)$
$= 25(-76)$
$\boxed{S_{50} = -1900}$

2. $a_1 = -8$, $d = -7$, & $a_n = -71$

$a_n = a_1 + (n-1)d$
$-71 = -8 + (n-1)(-7)$
$-71 = -8 - 7n + 7$
$-71 = -7n - 1$
$-70 = -7n$
$10 = n$

$S_{10} = \frac{10}{2}(-8 + -71)$
$= 5(-79)$
$\boxed{S_{10} = -395}$

3. $a_1 = 32$, $n = 27$, & $d = 3$

$S_{27} = \frac{27}{2}(2 \cdot 32 + (27-1)3)$
$S_{27} = 13.5(64 + 26 \cdot 3)$
$= 13.5(64 + 78)$
$= 13.5(142)$
$\boxed{S_{27} = 1917}$

4. $8 + 6 + 4 + \ldots + -10$

$a_1 = 8$
$a_n = -10$
$d = -2$
$n = ?$

$-10 = 8 + (n-1)(-2)$
$-10 = 8 - 2n + 2$
$-10 = 10 - 2n$
$-20 = -2n$
$10 = n$

$S_{10} = \frac{10}{2}(8 + -10)$
$S_{10} = 5(-2)$
$\boxed{S_{10} = -10}$

5. $16 + 22 + 28 + \ldots + 112$

$a_1 = 16$
$a_n = 112$
$d = 6$
$n = ?$

$112 = 16 + (n-1)6$
$112 = 16 + 6n - 6$
$112 = 22 + 6n$
$90 = 6n$
$15 = n$

$S_{15} = \frac{15}{2}(16 + 112)$
$= 7.5(128)$
$\boxed{S_{15} = 960}$

Find the first three terms of each arithmetic series described.

6. $a_1 = 12$, $a_n = 174$, and $S_n = 1767$

$S_n = \frac{n}{2}(a_1 + a_n)$
$1767 = \frac{n}{2}(12 + 174)$
$1767 = \frac{n}{2}(186)$
$1767 = 93n$
$n = 19$

$a_n = a_1 + (n-1)d$
$174 = 12 + (19-1)d$
$174 = 12 + 18d$
$162 = 18d$
$9 = d$

$\boxed{12 + 21 + 30 \text{ 1st 3 terms}}$

7. $a_1 = 80$, $a_n = -115$, and $S_n = -245$

$-245 = \frac{n}{2}(80 + -115)$
$-245 = \frac{n}{2}(-35)$
$-245 = -17.5n$
$14 = n$

$-115 = 80 + (14-1)d$
$-115 = 80 + 13d$
$-195 = 13d$
$-15 = d$

$\boxed{80 + 65 + 50 \text{ 1st 3 terms}}$

✖ SUMMATION/SIGMA NOTATION: $\boxed{\sum_{k=1}^{x} a_n = a_1 + a_2 + a_3 + \ldots + a_x}$

- $\sum$ is a symbol for S_n, which means to add up the <u>number</u> of terms expressed in the numbers above and below the symbol.
- The # of terms is found by this formula: $\boxed{\sum_{k=x}^{b} a_n \quad n = b - x + 1}$
- n = x: x is the term you <u>start</u> with (a_1). Find it by plugging x into the expression.
- a_n = the nth term formula, which is $a_n = a_1 + (n - 1)d = a_1 + dn - d$ Since $a_1 - d = a_0 \longrightarrow \boxed{a_n = dn + a_0}$
- Write in expanded form means to "do it the long way" instead of using the S_n formula.

✖ EXPRESS A SERIES IN SIGMA NOTATION:

In #4 on the front (8 + 6 + 4 + . . . + -10), we found that d = -2, -10 is the 10th term, so we put the #'s in the formula: $\sum_{k=x}^{b} a_n \longrightarrow \sum_{k=1}^{10} a_n$ $\quad a_n = a_1 + (n-1)d = 8 + (n - 1)(-2) = 8 + -2n + 2 = -2n + 10 \longrightarrow \boxed{\sum_{k=1}^{10} -2n+10}$

✖ Think: If y = mx + b, then d = m and b = a_0 . . . so for an arithmetic sequence: $\boxed{\sum_{k=1}^{b} dn + a_0}$

✖ **Examples done by me:**

A. Write in <u>expanded form</u> and find the sum.

$$\sum_{k=1}^{5} 2k-1 = 2(1)-1+2(2)-1+2(3)-1+2(4)-1+2(5)-1 = \boxed{25}$$

B. Find the sum using the summation formula. (Do not expand.)

$n = 5 - 1 + 1 = \underline{5}$, $a_1 = 2(1) - 1 = \underline{1}$, $a_n = 2(5) - 1 = \underline{9}$, therefore $S_5 = \frac{5}{2}(1 + 9) = 2.5(10) = \boxed{25}$

✖ **Examples done by you: Find the sum of each series.**

1. $\sum_{k=1}^{18} 2k - 7$

$a_1 = 2(1) - 7 = -5$
$a_{18} = 2(18) - 7 = 29$
$n = 18 - 1 + 1 = 18$

$\boxed{S_{18} = \frac{18}{2}(-5+29) = 216}$

2. $\sum_{k=5}^{25} k - 1$

$a_5 = 5 - 1 = 4$
$a_{25} = 25 - 1 = 24$
$n = 25 - 5 + 1 = 21$

$\boxed{S = \frac{21}{2}(4+24) = 294}$

3. $\sum_{k=10}^{75} 2k - 200$

$a_{10} = 2(10) - 200$
$= -180$
$a_{75} = 2(75) - 200$
$= -50$
$n = 75 - 10 + 1 = 66$

$\boxed{S = \frac{66}{2}(-180 + -50) = -7590}$

✖ **Write summation notation for each of the given series.**

4. -4, 0, 4, 8, 12, 16

$d = 4$, a_1, a_6

$a_n = -4 + (n-1)4$
$a_n = -4 + 4n - 4$
$= 4n - 8$

$\boxed{\sum_{k=1}^{6} 4n-8}$

5. 16 + 22 + 28 + . . . + 112 (#5 on the front) $d = 6$

a_1, a_{15} (from front n=15)

$a_n = 16 + (n-1)6$
$a_n = 16 + 6n - 6$
$a_n = 6n + 10$

$\boxed{\sum_{k=1}^{15} 6n+10}$