11-2 Arithmetic Sequences

ARITHMETIC SEQUENCE: a sequence of numbers where the following is true:

- Each term is found by **adding** a constant, called the **common difference**, to the previous term.
- When each point is graphed (n, a_n), it creates a discrete set of points, which form a linear function $f(x) = dx + a_0$ when connected.
- The equation of the function connecting the points is related to the terms of the sequence: the slope is the _____ difference (d) and the y-intercept is the _____ term (a_0).

COMMON DIFFERENCE:

$d = a_n - a_{n-1}$

ARITHMETIC n^{th} TERM:

$a_n = a_1 + (n-1)d$
$a_n = a_1 + dn - d$
$a_n = dn + a_0$

n is the term # you are finding, a_1 is the first term, and d is the common difference

*$a_0 = a_1 - d$

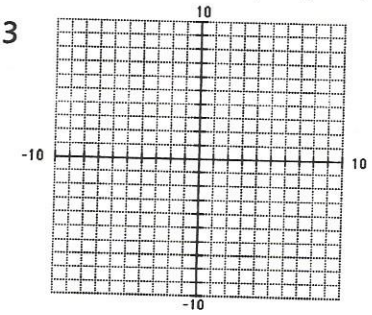
Example: Write an equation for the n^{th} term of the arithmetic sequence. Then find a_{10}.

1, 4, 7, 10, 13, 16, . . .

$d = a_n - a_{n-1} = 4 - 1 = 3$
$a_n = 3n - 2$

$a_n = a_1 + (n-1)d$
$a_n = 1 + (n-1)3$
$a_n = 1 + 3n - 3$

$\therefore a_n = 3n - 2$



Use the nth term equation or formula:

$a_n = 3n - 2$
$a_{10} = 3(10) - 2$
$a_{10} = 30 - 2$

$a_{10} = 1 + (10-1)(3)$
$a_{10} = 1 + (9)(3)$
$a_{10} = 1 + 27$

$\therefore a_{10} = 28$

Write an equation for the n^{th} term of the arithmetic sequence described. Then find a_n.

1. $a_1 = -5, d = 4, n = 9$

P1: $a_n = a_1 + (n-1)d$
$a_n = -5 + (n-1)4$
$a_n = -5 + 4n - 4$
$a_n = 4n - 9$

P2: $a_9 = 4(9) - 9$
$a_9 = 27$

2. $a_1 = 13, d = \frac{-5}{2}, n = 29$

P1: $a_n = a_1 + (n-1)d$
$a_n = 13 + (n-1)(-\frac{5}{2})$
$a_n = 13 - \frac{5}{2}n + \frac{5}{2} = -\frac{5}{2}n + \frac{31}{2}$

P2: $a_{29} = -\frac{5}{2}(29) + \frac{31}{2}$
$a_{29} = -57$

3. $a_1 = 3, d = -4, n = 6$

P1 ⇒ $a_n = a_1 + (n-1)d$
$a_n = 3 + (n-1)(-4)$
$a_n = 3 - 4n + 4$
$a_n = -4n + 7$

P2 ⇒ $a_6 = -4(6) + 7$
$a_6 = -17$

4. $a_1 = -5, d = \frac{1}{2}, n = 10$

P1 ⇒ $a_n = -5 + (n-1)(\frac{1}{2})$
$a_n = -5 + \frac{1}{2}n - \frac{1}{2}$
$a_n = \frac{1}{2}n - \frac{11}{2}$

P2 ⇒ $a_{10} = \frac{1}{2}(10) - \frac{11}{2}$
$a_{10} = -\frac{1}{2}$

Find the indicated term in each arithmetic sequence.

5. a_{15} for −3, 3, 9, ...

$a_1 = -3$, $d = 6$, $n = 15$, $a_n = ?$

$a_{15} = -3 + (15-1)6$
$a_{15} = -3 + (14)(6) = 81$

6. a_{19} for 17, 12, 7, ...

$a_1 = 17$, $d = -5$, $n = 19$

$a_{19} = 17 + (19-1)(-5)$
$a_{19} = -73$

7. a_{26} for $1, \frac{7}{3}, \frac{11}{3}, \ldots$

$a_1 = 1$, $d = \frac{4}{3}$, $n = 26$

$a_{26} = 1 + (26-1)\cdot\frac{4}{3}$
$a_{26} = \frac{103}{3}$

8. a_{35} for $17, 16\frac{2}{3}, 16\frac{1}{3}, \ldots$

$a_1 = 17$, $d = -\frac{1}{3}$, $n = 35$

$a_{35} = 17 + (35-1)(-\frac{1}{3})$
$a_{35} = \frac{17}{3}$

Example: Fill in the blank. 142 is the ____ th term of -3, 2, 7, . . .

$d = 5,\ 2-(-3) = 5,\ 7-2 = 5$
$a_1 = -3,\ a_n = 142$
$142 = -3 + (n-1)5$
$142 = -3 + 5n - 5$
$142 = 5n - 8$
$150 = 5n \qquad n = 30$

$\therefore$ 142 is the 30th term

Find the sequence number of the given term.

9. 97 is the _th term of –3, 1, 5, 9, …

$a_1 = -3$
$a_n = 97$
$d = 4$
$n = ?$

$97 = -3 + (n-1)\cdot 4$
$97 = -3 + 4n - 4$
$97 = 4n - 7$
$104 = 4n$
$n = 26$

10. -10 is the _th term of 14, 12.5, 11, 9.5,…

$a_1 = 14$
$a_n = -10$
$d = -1.5$
$n = ?$

$-10 = 14 + (n-1)\cdot -1.5$
$-10 = 14 - 1.5n + 1.5$
$-10 = -1.5n + 15.5$
$-25.5 = -1.5n$
$17 = n$

Example: Find the arithmetic means (missing terms) of the following sequence:

3, ____, ____, ____, 31

$a_1 = 3,\ a_5 = 31$
$a_n = a_1 + (n-1)\,d$
$a_5 = 3 + (5-1)\,d$
$31 = 3 + 4d$
$28 = 4d$, so $d = 7$

3, 3 + 7, 10 + 7, 17 + 7, 24 + 7

$\therefore$ 3, 10, 17, 24, 31

Find the missing terms in each arithmetic sequence.

11. 3, $\frac{26}{3}$, $\frac{43}{3}$, 20

$a_1 = 3$
$a_4 = 20$
$n = 4$
$d = ?$

$20 = 3 + (4-1)d$
$20 = 3 + 3d$
$17 = 3d$
$17/3 = d$

12. -16, -10, -4, 2, 8, 14

$a_2 = -10$
$a_6 = 14$
$n = 6$
$d = ?$

$14 = -10 + (6-2)d$
$14 = -10 + 4d$
$24 = 4d$
$6 = d$

$a_n = a_2 + (n-2)d$

13. 5, $\frac{37}{3}$, $\frac{59}{3}$, 27

$a_1 = 5$
$a_4 = 27$
$n = 4$
$d = ?$

$27 = 5 + (4-1)d$
$27 = 5 + 3d$
$22 = 3d$
$\frac{22}{3} = d$

14. -2.25, 4, 10.25, 16.5, 22.75, 29

$a_2 = 4$
$a_6 = 29$
$n = 6$
$d = ?$

$a_n = a_2 + (n-2)d$
$29 = 4 + (6-2)d$
$29 = 4 + 4d$
$25 = 4d$
$6.25 = d$

15. How many multiples of 11 are there between 13 and 384?

$a_1 = 22$
$a_n = 374$
$d = 11$
$n = ?$

22, 33, 44, . . . , 374

$374 = 22 + (n-1)\cdot 11$
$374 = 22 + 11n - 11$
$374 = 11 + 11n$
$363 = 11n$
$33 = n$