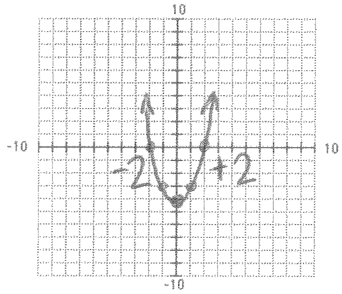
master

5-4 Complex Numbers

1. Solve $x^2 - 4 = 0$ by graphing



Solve by factoring:

$(x+2)(x-2)=0$

$x = \pm 2$

Solve using square roots:

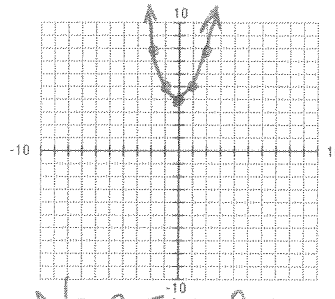
$x^2 - 4 = 0$

$x^2 = 4$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

2. Solve $x^2 + 4 = 0$ by graphing



NO REAL SOL.

Solve by factoring:

doesn't factor!

Solve using square roots:

$x^2 + 4 = 0$

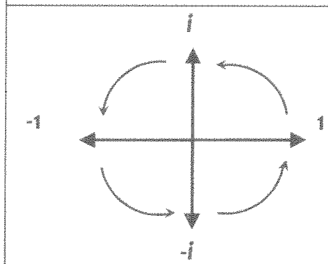
$x^2 = -4$

$\sqrt{x^2} = \sqrt{-4}$

$x = \pm\sqrt{-4} = \pm 2$

Equations like #2 above is what led mathematicians to DEFINE ***imaginary numbers.*** Imaginary numbers were first mentioned back in the 1st Century AD, but Leonhard Euler is the mathematician who introduced the symbol *i* to represent $\sqrt{-1}$ in the 1700's. Imaginary numbers "are non-real, but they are still numbers!" Since these numbers have been defined, we can now solve problems when they are involved.

Imaginary numbers can be thought of a ROTATION of a real number on the **number line**. Just like the negative numbers represent direction (right is positive, left is negative), i is a 90° rotation. Multiplying by i twice (i^2) gets you to -1 multiplying by i four times (i^4) gets you back to +1.



This is the "Argand plane" and it represents the powers of *i.*

EVALUATING i^n:

i^1	$\sqrt{-1} = i$	i^5	i
i^2	$\sqrt{-1} \bullet \sqrt{-1} = -1$	i^6	-1
i^3	$\sqrt{-1} \bullet \sqrt{-1} \bullet \sqrt{-1} = -1 \bullet \sqrt{-1} = -i$	i^7	$-i$
i^4	$\sqrt{-1} \bullet \sqrt{-1} \bullet \sqrt{-1} \bullet \sqrt{-1} = (-1)(-1) = 1$	i^8	1

See the pattern and find:

1. i^{12} 1	**2.** i^{15} 1 less than i^{16}! $-i$	**3.** i^{77} $i^{76} \cdot i = 1 \cdot i$ i	**4.** i^{132} $= (i^4)^{33} = 1^{33}$ 1

Imaginary Number *(i)* : because of the property above, the imaginary number also allows us to take the square root of negative numbers. i is definded as the principal square root of -1:

- $i = \sqrt{-1}$
- $i^2 = -1$

- *i* is NOT a variable.
- Use *i* as you would any constant.
- *i* is NOT a real number… it is called imaginary not because it does not exist, but because many mathematicians refused to believe in it at first. "Imaginary" was meant as a derogative term!!

Pure Imaginary Numbers: square roots of negative real #s.

- $\sqrt{-b^2} = \sqrt{b^2} \bullet \sqrt{-1} = b\,i$

Complex Number: a number that can be written in the form a + b*i*, where a and b are real #s and *i* is the imaginary unit.

Examples: $5 + i$ and $1 - 7i$

Complex Conjugate Pair: 2 solutions are complex numbers. One is a + b*i* and the other is a - b*i*

Examples: $\frac{-4 \pm 6i}{2} = -2 + 3i$ and $-2 - 3i$

SIMPLIFYING RADICALS: A radical is in simplest form when all exponents are positive, no perfect square factor or fraction is left under the radical, and no radicals are in the denominator.

Perfect Square Factorization

Since 24 has a factor that's perfect, then break it down as a product of factors and simplify rational factors.

$\sqrt{24} = \sqrt{4} \bullet \sqrt{6} = \boxed{2\sqrt{6}}$

Perfect Squares: 4, 9, 16, 25, 36, 49, . . .

$\sqrt{75}$

$\sqrt{25 \cdot 3}$

$5\sqrt{3}$

$\sqrt{\frac{7}{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{7 \cdot 3}}{\sqrt{3 \cdot 3}}$

$= \frac{\sqrt{21}}{3}$

SIMPLIFYING IMAGINARY NUMBERS: Principal root only

1. $\sqrt{-25}$

$5i$

2. $\sqrt{-28}$ $\sqrt{4}\sqrt{7}\sqrt{-1}$

$2i\sqrt{7}$

3. $\sqrt{-72}$ $\sqrt{-1} \cdot \sqrt{36}\sqrt{2}$

$6i\sqrt{2}$

4. $\sqrt{-24}$ $\sqrt{-1} \cdot \sqrt{4} \cdot \sqrt{6}$

$2i\sqrt{6}$

ADDING AND SUBTRACTING IMAGINARY NUMBERS:

5. $(3 + 5i) + (-2 + i)$

$1 + 6i$

6. $(-12 - 3i) - (-3 - 6i)$

$+3 + 6i$

$-9 + 3i$

7. $2i(3 + i) + (2 - 3i)$

$6i + 2i^2 + 2 - 3i$

$2i^2 = 2(-1) = -2$

$3i$

MULTIPLYING IMAGINARY NUMBERS: *There will never be an imaginary answer with a power above 1!*

8. $3i \bullet 8i$

$24i^2$

-24

9. $\sqrt{3} \bullet \sqrt{-12}$

$\sqrt{3} \cdot \sqrt{-1}\ \sqrt{3} \cdot \sqrt{4}$

$i \quad 3 \quad 2$

$6i$

10. $(\sqrt{2} + 3i)^2$

$(\sqrt{2} + 3i)(\sqrt{2} + 3i)$

$3i\sqrt{2} \quad 3i\sqrt{2}$

$2 + 6i\sqrt{2} + 9i^2$

9

$-7 + 6i\sqrt{2}$

11. $(7 + 4i)(7 - 4i)$

$49 - 16i^2$ -1

$49 - 16(-1)$

$49 + 16$

65

12. $-2i(3 - 2i)$

$-6i + 4i^2$

$-6i - 4$

$-4 - 6i$

13. $(5 - 2i)(\sqrt{5} + i)$

$5\sqrt{5} + 5i - 2i\sqrt{5} - 2i^2$

$5\sqrt{5} + 2 + 5i - 2i\sqrt{5}$

DIVIDING IMAGINARY NUMBERS:

14. $\frac{4 - 3i}{-3i} \cdot \frac{i}{i}$

$\frac{4i - 3i^2}{-3i^2}$

$\frac{3 + 4i}{3}$

15. $\frac{7}{1 + i} \cdot \frac{(1 - i)}{(1 - i)}$

$\frac{7 - 7i}{1 - i^2}$

$\frac{7 - 7i}{2}$

16. $\frac{11 + 6i}{2 - 3i} \cdot \frac{(2 + 3i)}{(2 + 3i)}$

$\frac{22 + 33i + 12i + 18i^2}{4 - 9i^2}$

$\frac{4 + 45i}{13}$

17. $\frac{2 + i}{\sqrt{3} - i} \cdot \frac{(\sqrt{3} + i)}{(\sqrt{3} + i)}$

$\frac{2\sqrt{3} + 2i + \sqrt{3}i + i^2}{3 - i^2}$

$\frac{2\sqrt{3} - 1 + 2i + i\sqrt{3}}{4}$

Gross!

SOLVING QUADRATIC EQUATIONS OVER THE SET OF COMPLEX NUMBERS:

22. $x^2 + 64 = 0$

$x^2 = -64$

$x = \pm\sqrt{-64}$

$x = \pm 8i$

23. $x^2 + 121 = 0$

$\sqrt{x^2} = -121$

$\sqrt{x^2} = \pm\sqrt{-121}$

$x = \pm 11i$

24. $x^2 + 3 = 0$

$x^2 = -3$

$\sqrt{x^2} = \pm\sqrt{-3}$

$x = \pm i\sqrt{3}$

25. $6x^2 + 40 = -2$

$6x^2 = -42$

$x^2 = -7$

$\sqrt{x^2} = \pm\sqrt{-7}$

$x = \pm i\sqrt{7}$

26. $-2x^2 - 8 = -2$

$-2x^2 = 6$

$x^2 = -3$

$\sqrt{x^2} = \pm\sqrt{-3}$

$x = \pm i\sqrt{3}$

27. $\frac{1}{3}x^2 = -15$

$x^2 = -45$

$\sqrt{x^2} = \pm\sqrt{-45}$

$x = \pm i\sqrt{45}$

$\sqrt{9 \cdot 5}$

$= \pm 3i\sqrt{5}$