Master E

9-2 Adding & Subtracting Rational Expressons

Recall the basics of adding or subtracting simple fractions... what must they have before you add or subtract?

a) $\frac{1}{8}+\frac{5}{8} = \frac{6}{8} = \frac{2\cdot 3}{2\cdot 4} = \frac{3}{4}$

b) $\frac{2}{5}-\frac{3}{7} = \frac{14}{35} - \frac{15}{35} = \frac{-1}{35}$

LCD 35

c) $\frac{8}{15}+\frac{5}{12} = \frac{32}{60} + \frac{25}{60} = \frac{57}{60} = \frac{19}{20}$

$3\cdot 5 \quad 3\cdot 4$

LCD: $3\cdot 5\cdot 4 = 60$

What happens when you bring in variables?

d) $\frac{2}{x}+\frac{3}{x} = \frac{5}{x}$

e) $\frac{3x}{x+2}-\frac{x+1}{x+2} = \frac{3x-(x+1)}{x+2} = \frac{3x-x-1}{x+2} = \frac{2x-1}{x+2}$

f) $\frac{5\cdot 4}{6a\cdot 4}-\frac{7\cdot 3a}{8\cdot 3a} = \frac{20}{24a} - \frac{21a}{24a}$

LCD $= 24a$

$\frac{-21a+20}{24a}$

Steps to Add and Subtract Rational Expressions:

1. Factor the denominators of each expression if they are different.
2. Find the Least Common Denominator (LCD).
3. For each rational expression, compare the denominator to the LCD and multiply each numerator by missing factors from LCD.
4. Combine the numerators of the rational expressions and put them over the LCD.
5. Simplify result by factoring numerator and canceling factors that are common with the denominator.

Detailed Examples to Illustrate the Proper Procedure:

Example A: $\frac{7}{6x}+\frac{11}{6x}=\frac{18}{6x}=\boxed{\frac{3}{x}}$

Example B: $\frac{2}{x-2}-\frac{3}{2-x}=\frac{2}{x-2}+\frac{3}{x-2}=\boxed{\frac{5}{x-2}}$

Example C: $\frac{6}{4x^2}+\frac{2}{5x}$

$4x^2 = 2\bullet 2\bullet x\bullet x$; $5x = 5\bullet x$

LCD: $2\bullet 2\bullet 5\bullet x\bullet x = 20x^2$

$=\frac{6\bullet 5}{20x^2}+\frac{2\bullet 4x}{20x^2}=\frac{30}{20x^2}+\frac{8x}{20x^2}=\frac{30+8x}{20x^2}$

$=\frac{2(4x+15)}{20x^2}=\boxed{\frac{4x+15}{10x^2}}$

Example D: $\frac{10}{x^2-5x-14}+\frac{2}{x-7}$

$(x-7)(x+2) \quad (x-7)$

LCD: $(x-7)(x+2)$

$=\frac{10}{(x-7)(x+2)}+\frac{2(x+2)}{(x-7)(x+2)}$

$=\frac{10+2(x+2)}{(x-7)(x+2)}=\frac{10+2x+4}{(x-7)(x+2)}$

$=\frac{2x+14}{(x-7)(x+2)}=\boxed{\frac{2(x+7)}{(x-7)(x+2)}}$

Example E: $\frac{x+1}{x^2+4x+4}-\frac{2}{x^2-4}$

$(x+2)^2 \quad (x+2)(x-2)$

LCD: $(x+2)^2(x-2)$

$=\frac{(x+1)(x-2)}{(x+2)^2(x-2)}-\frac{2(x+2)}{(x+2)^2(x-2)}$

$=\frac{(x+1)(x-2)-[2(x+2)]}{(x+2)^2(x-2)}=\frac{x^2-x-2-2x-4}{(x+2)^2(x-2)}$

$=\boxed{\frac{x^2-3x-6}{(x+2)^2(x-2)}}$

1-6: Simplify each expression completely.

1. $\frac{1}{5n}-\frac{3}{4}+\frac{7}{10n}$ LCD $= 20n$

$5\cdot n \quad 2\cdot 2 \quad 2\cdot 5\cdot n$

$\frac{1\cdot 4}{5n\cdot 4} = \frac{4}{20n}$

$-\frac{3\cdot 5n}{4\cdot 5n} = \frac{-15n}{20n}$

$\frac{7\cdot 2}{10n\cdot 2} = \frac{14}{20n}$

$\frac{4-15n+14}{20n}$

$\frac{-15n+18}{20n}$

2. $\frac{4}{a-3}+\frac{9}{a-5}$ LCD $= (a-3)(a-5)$

$\frac{4}{a-3} = \frac{4(a-5)}{(a-3)(a-5)} = \frac{4a-20}{(a-3)(a-5)}$

$\frac{9}{a-5} = \frac{9(a-3)}{(a-3)(a-5)} = \frac{9a-27}{(a-3)(a-5)}$

$\frac{4a-20+9a-27}{(a-3)(a-5)} = \frac{13a-47}{(a-3)(a-5)}$

3. $\frac{16}{x^2-16}+\frac{2}{x+4}$ LCD $= (x+4)(x-4)$

$(x+4)(x-4)$

$\frac{16}{(x+4)(x-4)} + \frac{2(x-4)}{(x+4)(x-4)}$

$\frac{16+2x-8}{(x+4)(x-4)} = \frac{2x+8}{(x+4)(x-4)}$

$\frac{2(x+4)}{(x+4)(x-4)} = \frac{2}{x-4}$

4. $\dfrac{5}{2x-12}-\dfrac{20}{x^2-4x-12}$ LCD$=2(x-6)(x+2)$

$2(x-6) \quad (x-6)(x+2)$

$$\frac{5(x+2)-20(2)}{2(x-6)(x+2)}$$

$$\frac{5x+10-40}{2(x-6)(x+2)}=\frac{5x-30}{2(x-6)(x+2)}$$

$$\frac{5\cancel{(x-6)}}{2\cancel{(x-6)}(x+2)}=\frac{5}{2(x+2)}$$

5. $\dfrac{2-5m}{m-9}+\dfrac{4m-5}{9-m}$

$$\frac{5m-2}{9-m}+\frac{4m-5}{9-m}$$

$$\frac{5m-2+4m-5}{9-m}$$

$$\frac{9m-7}{9-m}$$

6. $\dfrac{2x-2}{x^2+4x+3}-\dfrac{x-1}{x^2+5x+6}$

$(x+3)(x+1) \quad (x+3)(x+2)$

$$\frac{(2x-2)(x+2)}{(x+3)(x+1)(x+2)}-\frac{(x-1)(x+1)}{(x+3)(x+1)(x+2)}$$

$$\frac{2x^2+2x-4-(x^2-1)}{(x+3)(x+1)(x+2)}$$

$$\frac{2x^2+2x-4-x^2+1}{(x+3)(x+1)(x+2)}$$

$$\frac{x^2+2x-3}{(x+3)(x+1)(x+2)} \quad \frac{(x+3)(x-1)}{(x+3)(x+1)(x+2)}$$

$$=\frac{x-1}{(x+1)(x+2)}$$

Steps to Simplify Complex Fractions (Fractions on top of Fractions):

1. Write the numerator and denominator as single fractions by performing the indicated operation.
2. After simplifying the new numerator and denominator, multiply the numerator by the reciprocal of the denominator and simplify.
3. **OR:** Find the LCD of the numerator and denominator and multiply both by it. See examples!

Detailed Examples to help you Learn:

Example F: $\dfrac{\frac{x}{2}-5}{6+\frac{3}{x}}$

$$=\frac{\frac{x}{2}-\frac{10}{2}}{\frac{6x}{x}+\frac{3}{x}}=\frac{\frac{x-10}{2}}{\frac{6x+3}{x}} \quad \textbf{OR} \quad \frac{\left(\frac{x}{2}-5\right)\bullet 2x}{\left(6+\frac{3}{x}\right)\bullet 2x}=\frac{x^2-10x}{12x+6}$$

$$=\frac{\frac{x-10}{2}}{\frac{3(2x+1)}{x}}=\frac{x-10}{2}\bullet\frac{x}{3(2x+1)}$$

$$=\boxed{\frac{x(x-10)}{6(2x+1)}}$$

Example G: $\dfrac{\frac{4}{x^2-9}+\frac{2}{x-3}}{\frac{1}{x+3}+\frac{1}{x-3}}$

$$=\frac{\frac{4}{(x+3)(x-3)}+\frac{2}{(x-3)}}{\frac{1}{(x+3)}+\frac{1}{(x-3)}}=\frac{\frac{4+2(x+3)}{(x+3)(x-3)}}{\frac{(x-3)+(x+3)}{(x+3)(x-3)}} \quad \textbf{OR} \quad \frac{\left(\frac{4}{x^2-9}+\frac{2}{x-3}\right)}{\left(\frac{1}{x+3}+\frac{1}{x-3}\right)}\bullet\frac{(x+3)(x-3)}{(x+3)(x-3)}$$

$$=\frac{4+2(x+3)}{\cancel{(x+3)(x-3)}}\bullet\frac{\cancel{(x+3)(x-3)}}{(x-3)+(x+3)} \qquad =\frac{4+2(x+3)}{(x-3)+(x+3)}$$

$$=\frac{4+2(x+3)}{x-3+x+3}=\frac{4+2x+6}{2x}=\frac{2x+10}{2x}=\frac{2(x+5)}{2x}=\boxed{\frac{x+5}{x}}$$

7-8: Simplify each expression completely.

7. $\dfrac{\frac{r+6}{r}-\frac{1}{r+2}}{\frac{r^2+4r+3}{r^2+r}}$

$$\frac{\frac{(r+6)(r+2)-1(r)}{r(r+2)}}{\frac{(r+3)(r+1)}{r(r+1)}} \quad ☆$$

$$☆\ \frac{r^2+8r+12-r}{r(r+2)}=\frac{r^2+7r+12}{r(r+2)}$$

$$=\frac{\cancel{(r+3)}(r+4)}{\cancel{r}(r+2)}\bullet\frac{\cancel{r}\cancel{(r+1)}}{\cancel{(r+3)}\cancel{(r+1)}}$$

$$=\frac{r+4}{r+2}$$

8. $\dfrac{n+5-\frac{12}{n+1}}{\frac{n+9}{n+1}-\frac{5}{n}}$

$$\frac{\frac{(n+5)(n+1)-12}{n+1}}{\frac{n(n+9)-5(n+1)}{n(n+1)}}$$

$$\frac{\frac{n^2+6n+5-12}{n+1}}{\frac{n^2+9n-5n-5}{n(n+1)}}=\frac{\frac{n^2+6n-7}{n+1}}{\frac{n^2+4n-5}{n(n+1)}}=$$

$$=\frac{(n+7)\cancel{(n-1)}}{\cancel{(n+1)}}\bullet\frac{n\cancel{(n+1)}}{(n+5)\cancel{(n-1)}}=\frac{n(n+7)}{n+5}$$