Algebra II SOL Review: Expressions and Operations

Edited from VBCPS "Review Skills Notes Practice 2014"

SOL AII.1 The student, given rational, radical, or polynomial expressions, will
a) add, subtract, multiply, divide, and simplify rational algebraic expressions;

Hints and Notes

- To add or subtract: Must have a common denominator
- To multiply: Factor numerator, factor denominator, reduce/cancel common factors
- To divide: Flip the fraction ***after*** the division sign and use multiplication rules
- To simplify: factor numerator, factor denominator, reduce/cancel common factors
- Complex Fractions: Simplify numerator, simplify denominator, then divide

Key

PRACTICE A.1a

1. Which is equivalent to $\frac{x^2-4}{x^2-4x+4}$?

A $\frac{1}{x+1}$

[✓ B] $\frac{x+2}{x-2}$

C $\frac{1}{4x}$

D $\frac{1}{x+4}$

$= \frac{(x+2)\cancel{(x-2)}}{(x-2)\cancel{(x-2)}}$

$= \frac{x+2}{x-2}$

2. Which is equivalent to $\frac{6a+12}{a}\bullet\frac{a^3}{a+2}$

[✓ A] $6a^2$

B $\frac{6}{a^2}$

C $\frac{6(a+2)}{a}$

D $\frac{6a^2+24a+24}{a^4}$

$= \frac{6\cancel{(a+2)}}{a}\cdot\frac{a^3}{\cancel{(a+2)}}$

$= \frac{6a^3}{a} = 6a^2$

3. Which is equivalent to $\frac{{}^{2x\cdot}\,3x}{{}_{2x\cdot}\,7}+\frac{5y}{14x}$?

A $\frac{8y}{21}$

B $\frac{x^2}{14}$

[✓ C] $\frac{6x^2+5y}{14x}$

D $\frac{3x^2+5y}{14x}$

$= \frac{6x^2}{14x} + \frac{5y}{14x}$

$\frac{6x^2+5y}{14x}$

4. Which equivalent to $\frac{\frac{x}{x+7}}{\frac{x-9}{x+7}}$?

A -9

B $\frac{x^2-9x}{(x+7)^2}$

[✓ C] $\frac{x}{x-9}$

D $-\frac{1}{9}$

$= \frac{x}{(x+7)} \div \frac{(x-9)}{(x+7)}$

$= \frac{x}{\cancel{(x+7)}} \cdot \frac{\cancel{(x+7)}}{(x-9)}$

$= \frac{x}{x-9}$

5. Which is equivalent to $\frac{\frac{1}{x}-\frac{4}{y}}{\frac{2}{x}+\frac{5}{y}}$?

A $\frac{x-4y}{5x+2y}$

[✓ B] $\frac{y-4x}{2y+5x}$

C $\frac{x^2y^2}{(y-4x)(2y+5x)}$

D $2y^2-3xy-20x^2$

$= \frac{y-4x}{xy} \div \frac{2y+5x}{xy}$

$= \frac{y-4x}{\cancel{xy}} \cdot \frac{\cancel{xy}}{2y+5x}$

$= \frac{y-4x}{2y+5x}$

6. Which is equivalent to $\frac{(a+b)^{\cancel{3}}}{\underset{9}{\cancel{18}}}\bullet\frac{\overset{1}{\cancel{2}}}{\underset{1}{\cancel{(a+b)^2}}}$?

[✓ A] $\frac{a+b}{9}$

B $\frac{(a+b)^2}{9}$

C $\frac{(a+b)^5}{36}$

D $18a+9b$

$\frac{(a+b)}{9}$

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SOL AII.1 The student, given rational, radical, or polynomial expressions, will

- **b) add, subtract, multiply, divide, and simplify radical expressions containing rational numbers and variables, and expressions containing rational exponents;**
- **c) write radical expressions as expressions containing rational exponents and vice versa**

HINTS AND NOTES $\sqrt[b]{x^a} = x^{\frac{a}{b}}$, where ***a*** is the power and ***b*** is the index/root.

- To add or subtract radicals: Radicands and indices/roots must be the same. You may only add like radicals.
- Always simplify the radical completely.
- Pay attention to the index. Not every radical is a square root.

Key

PRACTICE AII.1bc

1. Which expression is equivalent to $\sqrt[3]{a^2}$?

A $a^{\frac{3}{2}}$
[✓ B] $a^{\frac{2}{3}}$
C $a^{\frac{1}{6}}$
D a^{6}

2. Which is equivalent to $\sqrt[3]{8x^6}$?

A 2
B $2x$
[✓ C] $2x^2$
D $2x^3$

3. Which is equivalent to $16^{\frac{3}{4}}$?

A 4
[✓ B] 8
C 12
D 32

$(\sqrt[4]{16})^3 = 2^3 = 8$

4. Which is equivalent to $a^{\frac{1}{2}}b^{\frac{3}{4}}$?

A ab^3
B $\sqrt{ab^3}$
C $\sqrt[3]{a^2b^4}$
[✓ D] $\sqrt[4]{a^2b^3}$

$= a^{\frac{2}{4}} b^{\frac{3}{4}}$

$= \sqrt[4]{a^2b^3}$

5. Which is equivalent to $2\sqrt{12}+3\sqrt{3}$?

A $16\frac{1}{2}$
B $5\sqrt{15}$
[✓ C] $7\sqrt{3}$
D $7\sqrt{6}$

$= 2\sqrt{4\cdot 3} + 3\sqrt{3}$

$= 2\cdot 2\sqrt{3} + 3\sqrt{3}$

$= 4\sqrt{3} + 3\sqrt{3}$

$= 7\sqrt{3}$

6. What is the simplest form of $\sqrt{72x^3} - 5x\sqrt{2x}$?

[✓ A] $x\sqrt{2x}$
B $\sqrt{2x}$
C $2x\sqrt{x}$
D $x^2\sqrt{2x}$

$= \sqrt{36\cdot 2\cdot x^2\cdot x} - 5x\sqrt{2x}$

$= 6x\sqrt{2x} - 5x\sqrt{2x}$

$x\sqrt{2x}$

7. What is the value of $\left(\frac{5^5}{2^5}\right)^{-1/5}$?

A $\frac{5}{2}$
B $\frac{25}{4}$
[✓ C] $\frac{2}{5}$
D $\frac{4}{25}$

$= \left(\frac{2^5}{5^5}\right)^{\frac{1}{5}} = \frac{\sqrt[5]{2^5}}{\sqrt[5]{5^5}}$

$= \frac{2}{5}$

8. Which is equivalent to the expression $\sqrt[3]{16} + 3\sqrt[3]{54} - 2\sqrt[3]{81}$

[✓ A] $11\sqrt[3]{2} - 6\sqrt[3]{3}$
B $11\sqrt[3]{2} - 2\sqrt[3]{3}$
C $2\sqrt[3]{2}$
D $5\sqrt[3]{2} - 6\sqrt[3]{3}$

$= \sqrt[3]{8\cdot 2} + 3\sqrt[3]{27\cdot 2} - 2\sqrt[3]{27\cdot 3}$

$= 2\sqrt[3]{2} + 3\cdot 3\sqrt[3]{2} - 2\cdot 3\sqrt[3]{3}$

$= 2\sqrt[3]{2} + 9\sqrt[3]{2} - 6\sqrt[3]{3}$

$= 11\sqrt[3]{2} - 6\sqrt[3]{3}$

SOL AII.1 The student, given rational, radical, or polynomial expressions, will
d) factor polynomials completely

HINTS AND NOTES

- Always look for a greatest common factor first: $xy + xw = x(y + w)$
- Look for patterns:

$a^2 - b^2 = (a + b)(a - b)$ $\quad$ $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
$a^2 + 2ab + b^2 = (a + b)^2$ $\quad$ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

To work backward to find factors: Multiply answer choices.

Key

PRACTICE AII.1d

1. Which is a factored form of $9x^2 - 25$?

[✓ **A**] $(3x-5)(3x+5)$
B $(3x-5)^2$
C $(3x+5)^2$
D $(9x-25)^2$

2. Which is a factor of $16x^2 - 1$?

$= (4x+1)(4x-1)$

A $(x-1)$
[✓ **B**] $(4x+1)$
C $(8x-1)$
D $4x$

3. Which is a factor of $x^2 - 2x - 15$?

$= (x-5)(x+3)$

A $(x-3)$
B $(x-15)$
[✓ **C**] $(x+3)$
D $(x+5)$

4. Which is a factor of $6a^2 + 5ab - 6b^2$?

[✓ **A**] $(2a+3b)$
B $(2a-3b)$
C $(3a+2b)$
D $(3a-3b)$

$(2a+3b)(3a-2b)$

5. Which is the factored form of $8x^3 + 1$?

A $(2x-1)(4x^2-2x+1)$
B $(2x-1)(4x^2+2x-1)$
[✓ **C**] $(2x+1)(4x^2-2x+1)$
D $(2x+1)(4x^2+2x-1)$

6. Which is the factored form of $1 - y^3$?

A $(1-y)(1-y-y^2)$
[✓ **B**] $(1-y)(1+y+y^2)$
C $(1+y)(1-y-y^2)$
D $(1+y)(1+y+y^2)$

7. Which represents the complete factorization of $4x^2 - 14x - 8$?

A $2(2x-1)(x+4)$ $= 2(2x^2-7x-4)$
B $2(2x+4)(x-1)$ $= 2(2x+1)(x-4)$
[✓ **C**] $2(2x+1)(x-4)$
D $2(2x-1)(x-4)$

8. Given the area of a rectangle being $2x^2 + 5x - 12$, Which of the following could represent the length of one side of the rectangle? (i.e. factor!) $A = lw$

$(2x-3)(x+4)$

A $2x+3$
[✓ **B**] $2x-3$
C $x-4$
D $x+12$

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SOL AII.3 The student will perform operations on complex numbers, express the results in simplest form using patterns of the powers of *i*, and identify field properties that are valid for the complex numbers.

Key

HINTS and NOTES: Graphics Calculator TIPS

- Use the i button on the calculator.
- Remember to include your parentheses. If $\frac{2+i}{3+i}$, then $(2+i)\div(3+i)$

Remember

$i=\sqrt{-1}$ *or just* i

$i^2=-1$

$i^3=-i$

$i^4=1$

The sequence repeats itself:

$i=i$	$i^5=i$	$i^9=i$
$i^2=-1$	$i^6=-1$	$i^{10}=-1$
$i^3=-i$	$i^7=-i$	$i^{11}=-i$
$i^4=1$	$i^8=1$	$i^{12}=1$

PRACTICE AII.3

1. **Which expression is equivalent to**

$$(6+2i)-(4+3i)$$

[✓ **A**] $2-i$
B $2+i$
C $2+5i$
D $10-i$

$6+2i-4-3i$

$2-i$

2. **Which is equivalent to**

$$(4-2i)(5+3i)$$

A 26
B 12
C $14+2i$
[✓ **D**] $26+2i$

$=20+12i-10i-6i^2$

$-6(-1)$

$=26+2i$

3. **Which is equivalent to** $(4-3i)^2$ **?**

A 25
B $25-2i$
C 7
[✓ **D**] $7-24i$

$=(4-3i)(4-3i)$

$=16-24i+9i^2$

$=16-24i-9$

$=7-24i$

4. **Which is equivalent to** $(3+2i)(2+5i)$ **?**

[✓ **A**] $-4+19i$
B $16+19i$
C $6+29i$
D $6-10i$

$6+15i+4i+10i^2$

$6+19i-10$

$-4+19i$

5. **Which is equivalent to** $\frac{5+i}{1+3i}$ **?**

A $\frac{4-8i}{5}$
[✓ **B**] $\frac{4-7i}{5}$
C $\frac{1-7i}{5}$
D $\frac{-1-7i}{4}$

$=\frac{(5+i)}{(1+3i)}\cdot\frac{(1-3i)}{(1-3i)}$

$=\frac{5-15i+i-3i^2}{1-9i^2}$

$=\frac{8-14i}{10}=\frac{4-7i}{5}$

6. **Which is equivalent to**

$$\sqrt{3}\bullet\sqrt{-3}$$

[✓ **A**] $3i$
B $-3i$
C 9
D $9i$

$=\sqrt{3}\cdot i\sqrt{3}$

$=3i$

7. **What number does** i^{24} **equal?**

A i
B -1
C $-i$
[✓ **D**] 1

$(i^4)^6=1^6=1$

TEI (Technology Enhanced Item): Free-Response - For free-response questions, type your answer in the box. Be sure your answer is in appropriate form - simplest fraction, decimal, etc.

8. **Simplify the following expression,** $(4+2i)(4-2i)$**. Type your answer in the box.**

$=16-4i^2$

$=16+4$

$\boxed{20}$