11-3 Geometric Series

Name_______________ Date_______________ Block_______

✖ **GEOMETRIC SERIES**: the sum of the terms of a geometric sequence.

✖ **SUM OF A GEOMETRIC SEQUENCE**: $a_1 + a_2 + \ldots + a_n \longrightarrow \boxed{S_n = \frac{a_1(1-r^n)}{(1-r)}, \text{ where } r \neq 1}$

- **Notation:** S_n: the Sum of n terms; n: the # of terms; a_1: the 1st term; a_n: the nth (last) term

Remember: $\boxed{a_n = a_1 r^{(n-1)}}$ is how you find the n^{th} term & $\boxed{r = \frac{a_n}{a_{n-1}}}$ is how you find the common ratio!

✖ **Examples for you to see:**

A. Find S_{10} of:

$7 + 14 + 28 + \ldots$

Since $\frac{14}{7} = 2$ and $\frac{28}{14} = 2$, then $r = 2$

$S_{10} = \frac{7(1-(2)^{10})}{(1-2)} = \frac{-7161}{-1}$

$\boxed{S_{10} = 7{,}161}$

B. Find the sum using the S_n formula.

$\sum_{k=1}^{12} 3^{k-1}$

$n = 12 - 1 + 1 = 12$, $r = 3$, $a_1 = 3^{1-1} = 3^0 \ldots$ so $a_1 = 1$!

$S_{12} = \frac{1(1-(3)^{12})}{(1-3)} = \frac{-531{,}440}{-2} = 265{,}720$

$\boxed{S_{12} = 265{,}720}$

✖ **Examples for you to do: Find the sum of each geometric series.**

1. $a_1 = 3, n = 4, r = \frac{1}{3}$

$S_4 = \frac{a_1(1-r^n)}{1-r}$

$= \frac{3(1-(\frac{1}{3})^4)}{1-\frac{1}{3}}$

$= \frac{40}{9}$

2. $a_1 = 100, n = 5, \& r = -\frac{1}{2}$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$= \frac{100(1-(\frac{1}{2})^5)}{1-\frac{1}{2}}$

$= \frac{775}{4}$

3. $a_3 = 20, a_6 = 160, \& n = 8$

find r: $160 = 20 \cdot r^{6-3}$

$8 = r^3$

$2 = r$

find a_1: $20 = a_1 \cdot 2^{3-1}$

$20 = a_1 2^2$

$20 = a_1 \cdot 4$

$5 = a_1$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$= \frac{5(1-2^8)}{1-2}$

$= \frac{5(1-256)}{1-2}$

$\boxed{= 1275}$

4. $6 + 18 + 54 + \ldots$ to 6 terms

$a_1 = 6$

$r = 3$

$n = 6$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$= \frac{6(1-3^6)}{1-3}$

$\boxed{= 2184}$

Find a_1 for each geometric series described.

5. $S_n = 183, r = -3, n = 5$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$183 = \frac{a_1(1-(-3)^5)}{1-(-3)}$

$183 = \frac{a_1(1-(-243))}{4}$

$732 = a_1(1+243)$

$732 = 244a_1$

$\boxed{a_1 = 3}$

6. $S_n = 1705, r = 4, n = 5$

$1705 = \frac{a_1(1-4^5)}{1-4}$

$1705 = \frac{a_1(1-1024)}{-3}$

$-5115 = a_1(-1023)$

$\boxed{a_1 = 5}$

SUMMATION/SIGMA NOTATION:

$$\boxed{\sum_{k=1}^{x} a_n = a_1 + a_2 + \ldots + a_x}$$

$$\boxed{\sum_{k=x}^{b} a_n = a_1(r)^{(n-1)}}$$

Remember: $\boxed{n = b - x + 1}$

Examples for you to see:

A. Write in expanded form and find the sum.

$$\sum_{k=1}^{6} 2^k = 2+4+8+16+32+64 = 126$$

B. Find the sum using the summation formula. (Do not expand.)

$$\sum_{k=1}^{6} 2^k = \frac{2(1-(2)^6)}{(1-2)} = \frac{(-126)}{-1} = 126$$

Examples for you to do: Find the sum of each geometric series.

1. $\sum_{k=1}^{10} 3^{k-1}$

$a_1 = 3^{1-1} = 3^0 = 1$

$r = 3$

$n = 10 - 1 + 1 = 10$

$S_n = \frac{a_1(1-r^n)}{1-r}$

$= \frac{1(1-3^{10})}{1-3} = 29524$

2. $\sum_{k=3}^{7} 3 \bullet 2^{k-1}$

$a_3 = 3 \cdot 2^{3-1} = 12$

$r = 2$

$n = 7 - 3 + 1 = 5$

$S_n = \frac{a_3(1-r^n)}{1-r}$

$= \frac{12(1-2^5)}{1-2}$

$\boxed{= 372}$

3. $\sum_{k=4}^{8} 2\left(\frac{1}{2}\right)^{k-1}$

$a_4 = 2\left(\frac{1}{2}\right)^{4-1} = \frac{1}{4}$

$r = \frac{1}{2}$

$n = 8 - 4 + 1 = 5$

$S_n = \frac{a_4(1-r^n)}{1-r}$

$= \frac{\frac{1}{4}(1-(\frac{1}{2})^5)}{1-\frac{1}{2}}$

$= \frac{31}{64}$

Write Summation notation for each series described.

4. $6 + 18 + 54 + \ldots$ to 6 terms

$r = 3$

$a_n = 6 \cdot 3^{n-1}$

$n = 6$

$\sum_{k=1}^{6} 6 \cdot 3^{k-1}$

5. $\frac{1}{4} + \frac{1}{2} + 1 + \ldots$ to 10 terms

$r = 2$

$a_1 = \frac{1}{4}$

$n = 10$

$a_n = \frac{1}{4} \cdot \left(\frac{1}{2}\right)^{n-1}$

$\sum_{k=1}^{10} \frac{1}{4} \cdot \left(\frac{1}{2}\right)^{k-1}$