6-3 Polynomial Functions

**Polynomial in**

**One Variable**

A polynomial of degree *n* in one variable *x* is an expression of the form

*an xn* + *an*–1*xn* – 1 + *…* + *a*2 *x*2 + *a*1*x* + *a*0, *where the coefficients an*–1*, an*–2*, an*–3*, …, a*0 *represent real numbers, an is not zero, and n represents a nonnegative integer.*

♥The **degree of a polynomial** in one variable is the greatest exponent of its variable. ♥The **leading coefficient** is the coefficient of the term with the highest degree.

**Polynomial**

**Function**

A polynomial function of degree *n* can be described by an equation of the form

*P*(*x* ) = *anxn* + *an*–1*xn*–1+ … + *a*2*x*2+ *a*1 *x* + *a*0, where the coefficients *an*–1, *an*–2, *an*–3, …, *a*0represent real numbers, *an* is not zero, and *n* represents a nonnegative integer.

 ♥ The exponents are all whole numbers.

 ♥ The coefficients and constant are all real numbers.

General/Standard form: The terms are written in descending order

**Evaluating a function:** If you know an element in the domain of any polynomial function,

 you can find the corresponding value in the range.

**Example 1**

**What are the degree and leading coefficient of 3*x*2 – 2*x*4 – 7 + *x*3?**

Rewrite the expression so the powers of *x* are in decreasing order.

–2*x*4 + *x*3 + 3*x*2 – 7

This is a polynomial in one variable. The degree is 4, and the leading coefficient is –2.

**Lesson 6-3**

Original function

Replace x with a2 – 1. Evaluate.

Simplify.

*g*(*x*) = *x*2 + 3*x* – 4

*g*(*a*2 – 1) = (*a*2 – 1)2 + 3(*a*2 – 1) – 4

= *a*4 – 2*a*2 + 1 + 3*a*2 – 3 – 4

= *a*4 + *a*2 – 6

**Find *g*(*a*2 – 1) if *g*(*x*) = *x*2 + 3*x* – 4.**

**Example 3**

Original function

Replace x with –5. Evaluate.

Simplify.

*f* (*x*) = *x*3 + 2*x*2 – 10*x* + 20

*f* (–5) = (–5)3 + 2(–5)2 – 10(–5) + 20

= –125 + 50 + 50 + 20

= –5

**Example 2**

**Find *f*(–5) if *f* (*x*) = *x*3 + 2*x*2 – 10*x* + 20.**

**State the degree and leading coefficient of each polynomial in one variable. If it is not a polynomial in one variable, explain why.**

**3.** 4*x*6 + 6*x*4 + 8*x*8 – 10*x*2 + 20

**2.** 100 – 5*x*3 + 10*x*7

**1.** 3*x*4 + 6*x*3 – *x*2 + 12

**6.** $\frac{x^{2}}{18}-\frac{x^{6}}{25}+\frac{x^{3}}{36}-\frac{1}{72}$

**5.** 8*x*3 – 9*x*5 + 4*x*2 – 36

**4.** 4*x*2 – 3*xy* + 16*y*2

**Given f(x) = 3x2 – 9 and g(x) = 4*x*3 – 3*x*2 + 2*x* – 1, find each value.**

 **7.** f(3a) **8.** g(-4) **9.** f(x + 2)

**Graphs of Polynomial Functions:** The general shapes of the graphs of polynomial functions shows the maximum number of times the graph of each function may intersect the x-axis. This is the same number as the degree of the polynomial.



**End Behavior of a Polynomial Function:**







**For each graph, do the following:**

**a. describe the end behavior,**

**b. determine whether it represents an odd-degree or an even-degree function, and**

**c. state the number of real zeroes.**

**10. 11. 12.**