11-3 Geometric Sequences

GEOMETRIC SEQUENCE: a sequence of numbers where the following is true:

- each term is found by **multiplying** a constant, called the **common ratio**, to the previous term.
- When each point is graphed (n, a_n), it creates a discrete set of points, which form an exponential function $f(x) = a_1 r^{n-1}$ when connected.
- The equation of the function connecting the points is related to the terms of the sequence: the base is the common ratio (r) and the y-intercept is the first term (a_1) divided by r.

COMMON RATIO:

$r = \frac{a_n}{a_{n-1}}$

GEOMETRIC n^th TERM:

$a_n = a_1 \bullet r^{n-1}$ n is the term # you are finding,

$a_n = a_0 \bullet r^n$ a_1 is the first term, and r is the common ratio

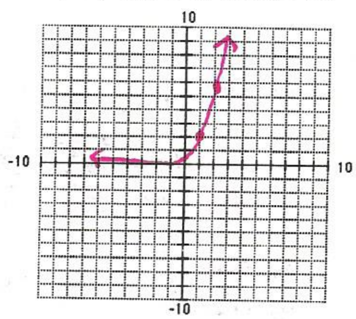
Example: Write an equation for the nth term of the sequence. Then find a_8.

2, 6, 18, 54, 162, . . .

Since $\frac{6}{2} = 3$, then r = 3

$a_n = a_1 \bullet r^{n-1}$

$\boxed{a_n = 2 \bullet (3)^{n-1}}$



Use the nth term equation:

$a_n = a_1 \bullet r^{n-1}$

$a_8 = 2(3)^{8-1}$

$a_8 = 2(3)^7$

$a_8 = 2(2187)$

$a_8 = 4{,}374$

CALCULATOR HINT: To find the 8th term, type in 2 and multiply by 3 to get the second term. Multiply the result by 3 again to get the third term. Push ENTER again to get the fourth term. Keep pushing ENTER until you find the 8th term.

Write an equation for the nth term of each sequence described. Then find a_n.

1. $a_1 = 5, n = 4, r = 3$

eq: $a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = 5 \cdot 3^{n-1}}$

find $a_n = a_4$

$a_4 = 5 \cdot 3^{4-1}$
$= 5 \cdot 3^3$
$= 5 \cdot 27$
$\boxed{a_4 = 135}$

2. $a_4 = 20, n = 6, r = -3$

eq: $a_n = a_1 \cdot r^{n-1}$

or $a_n = a_4 \cdot r^{n-4}$

$\boxed{a_n = 20 \cdot (-3)^{n-4}}$

find $a_n = a_6$

$a_6 = 20 \cdot (-3)^{6-4}$
$= 20 \cdot (-3)^2$
$= 20 \cdot 9$
$\boxed{= 180}$

3. $a_1 = -4, n = 6, r = -2$

eq: $a_n = a_1 \cdot r^{n-1}$

$\boxed{a_n = -4 \cdot (-2)^{n-1}}$

find $a_n = a_6$

$a_6 = -4 \cdot (-2)^{6-1}$
$= -4(-2)^5$
$= -4(-32)$
$\boxed{= 128}$

4. $a_6 = 8, n = 12, r = \frac{1}{2}$

eq: $a_n = a_1 \cdot r^{n-1}$

$a_n = a_6 \cdot r^{n-6}$

$\boxed{a_n = 8 \cdot (\frac{1}{2})^{n-6}}$

find $a_n = a_{12}$

$a_{12} = 8 \cdot (\frac{1}{2})^{12-6}$

$\boxed{= \frac{1}{8}}$

5. $a_3 = 9, r = -3, n = 7$

eq: $a_n = a_1 \cdot r^{n-1}$

$a_n = a_3 \cdot r^{n-3}$

$\boxed{a_n = 9 \cdot (-3)^{n-3}}$

find $a_n = a_7$

$a_7 = 9 \cdot (-3)^{7-3}$
$= 9 \cdot (-3)^4$
$\boxed{= 729}$

6. $a_3 = \frac{3}{8}, r = \frac{1}{2}, n = 6$

eq: $a_n = a_3 \cdot r^{n-3}$

$\boxed{a_n = \frac{3}{8} \cdot (\frac{1}{2})^{n-3}}$

$a_6 = \frac{3}{8} \cdot (\frac{1}{2})^{6-3}$
$= \frac{3}{8}(\frac{1}{2})^3$
$\boxed{= \frac{3}{64}}$

Find the first four terms of each geometric sequence described.

7. $a_1 = -6,\ r = \frac{-2}{3}$

$-6(-\frac{2}{3})$ $4(-\frac{2}{3})$ $-\frac{8}{3}(-\frac{2}{3})$

$4, -\frac{8}{3}, \frac{16}{9}, -\frac{32}{27}$

8. $a_1 = 2,\ r = \sqrt{3}$

$2, 2\sqrt{3}, 6, 6\sqrt{3}$

9. $a_1 = \frac{-5}{2},\ r = 2$

$-\frac{5}{2}, -5, -10, -20$

Example: Find the geometric means (missing terms) of the following sequence.

3, ____, ____, 375

First find r, then the missing terms.

$a_1 = 3,\ a_4 = 375$, so use $n = 4$

$a_4 = 3(r)^{4-1}$

$375 = 3(r)^3$

$125 = r^3 \quad r = \sqrt[3]{125} \quad r = 5$

3, 3(5), 15(5), 75(5)

$\boxed{\therefore 3,\ \underline{15},\ \underline{75},\ 375}$

Find the missing geometric means.

10. $\underline{\frac{2}{9}}, \underline{\frac{2}{3}}, 2, \underline{6}, \underline{18}, 54$

$a_3 = 2$
$a_6 = 54$
$n = 6$
$r = ?$

$a_n = a_3 \cdot r^{n-3}$

$54 = 2 \cdot r^{6-3}$

$54 = 2r^3$

$27 = r^3$

$3 = r$

11. 32, ____, ____, ____, 162

ans 1: 32 48 72 108 162 $r = \frac{3}{2}$

ans 2 32 −48 72 −108 162 $r = -\frac{3}{2}$

$a_1 = 32$
$a_5 = 162$
$n = 5$
$r = ?$

$a_n = a_1 \cdot r^{n-1}$

$162 = 32 r^{5-1}$

$\frac{162}{32} = r^4$

$\frac{81}{16} = r^4 \quad r = \pm\frac{3}{2}$ two ans!

12. 3, $\underline{-12}$, $\underline{48}$, -192, $\underline{768}$

$a_1 = 3$
$a_4 = -192$
$n = 4$
$r = ?$

$a_n = a_1 \cdot r^{n-1}$

$-192 = 3 \cdot r^{4-1}$

$-192 = 3r^3$

$-64 = r^3$

$-4 = r$

13. $\underline{\frac{4}{3}}, \frac{4}{9}, \underline{\frac{4}{27}}, \underline{\frac{4}{81}}, \frac{4}{243}$

$a_2 = \frac{4}{9}$
$a_5 = \frac{4}{243}$
$n = 5$
$r = ?$

$a_5 = a_2 r^{n-2}$

$\frac{4}{243} = \frac{4}{9} r^{5-2}$

$\frac{4}{243} = \frac{4}{9} r^3$

$\frac{1}{27} = r^3 \quad r = \frac{1}{3}$

14. Each foot of water screens out 60% of light above. What percent of the light *remains* after passing through 5 feet of water?

40% $r = .4$

$a_5 = (.4)^5 = .01024$ 1.024% remains