2-2 Logic, Venn Diagrams, & Truth Tables

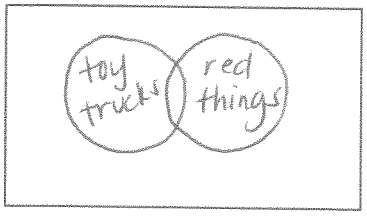
Venn diagram: a picture with circles that is used to represent a conditional statement or the relationship of sets of objects. The sizes of the circles do not indicate the relative sizes of the sets.

Quantifiers: words with special meanings in logic that imply numeric value

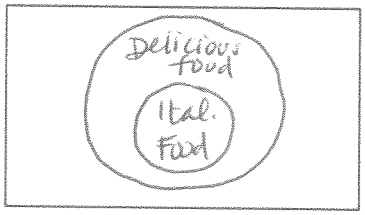
- We represent their meaning through the use of venn diagrams.
- Some quantifier words are: all, some, none, most, atleast

Examples:

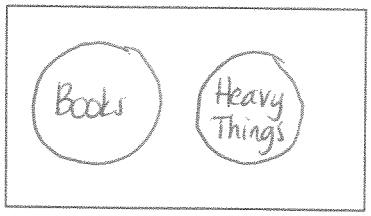
1. Some toy trucks are red.



2. All Italian food is delicious.



3. None of the books were heavy

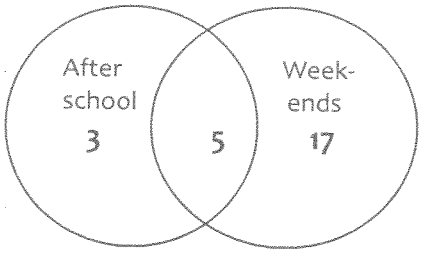


The Venn Diagram below shows the number of students in the band who work after school or on weekends.

4. How many students work after school and on weekends? 5
   What led you to this conclusion?

5. How many students work after school or on weekends? 25
   What led you to this conclusion? 3+17+5

6. From the Venn diagram, how many students are in the band?
   Explain your answer.



* we don't know how many. ✓ can't determine, but at least 25!

TRUTH VALUES		
Term	**Definition**	**Example/Notation**
Statement	A statement is any sentence that is either true or false.	*p*: Virginia Beach is in Florida F *q*: Chicago is on Lake Michigan. T
Truth Value	The truth value of a statement is either true or false	T or F
Negation	A statement that has the opposite meaning and opposite truth value of an original statement.	~*p* (Read: *not p*) ~*p*:. Virginia Beach is not in Florida. Since p is F, then ~p is T
Compound Statement	Two or more statements joined by the word AND or OR	p and q p or q
Conjunction	A compound statement formed by joining two or more statements using the word *and*. **The conjunction p ∧ q is true only when both p and q are true.**	$p \wedge q$ (Read: *p and q*) Virginia Beach is in Florida (F) and Chicago is on Lake Michigan (T). p ∧ q: T or (F)
Disjunction	A compound statement formed by joining two or more statements using the word *or*. **The disjunction p ∨ q is true if only one (p or q) is true.**	$p \vee q$ (Read: *p or q*) Virginia Beach is in Florida (F) or Chicago is on Lake Michigan (T). p ∨ q: (T) or F

1-4: Use the given statements to complete each compound statement. Then find its truth value.

p: An elephant is a mammal.
q: A square has four right angles.

r: A diameter of a circle is twice the radius.
s: A rectangle has four equal sides.

1. $p \wedge q$

An elephant is a mammal (T) and a square has 4 right angles. T

(T) or F

3. $r \vee s$

A diameter (T) of a circle is twice the radius or a rectangle has 4 equal sides. F

(T) or F

2. $\sim p \wedge q$

An elephant is not a mammal F and a square has 4 right angles. T

T or (F)

4. $\sim r \vee \sim s$

A diameter of a circle (F) is not twice the radius or a rectangle does not have 4 equal sides. F

(T) or F

Example of a Truth Table of $\sim p \wedge q$:

p	q	~p	~p ∧ q
T	T	F	F
T	F	F	F
F	T	T	T
F	F	T	F

AND

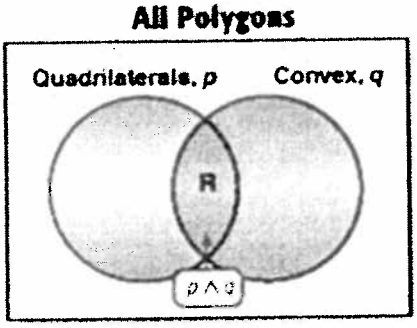
Example of a Truth Table of $p \wedge \sim q$:

p	q	~q	p ∧ ~q	p ∨ ~q
T	T	F	F	T
T	F	T	T	T
F	T	F	F	F
F	F	T	F	T

Conjunctions can be illustrated with Venn Diagrams:

p and *q*: A rectangle is a quadrilateral, and a rectangle is convex.

- The Venn diagram shows that a rectangle (R) is located in the *intersection* of the set of quadrilaterals and the set of convex polygons. In other words, rectangles must be in the set containing quadrilaterals *and* in the set of convex polygons.



Disjunctions can also be illustrated with Venn Diagrams:

- In the Venn diagram, the disjunction is represented by the *union* of the two sets. The union includes all polygons that are quadrilaterals, convex, *or* both.

The disjunction includes these three regions:

- $p \wedge \sim q$ quadrilaterals that are *not* convex
- $\sim p \wedge q$ convex polygons that are *not* quadrilaterals
- $p \wedge q$ polygons that are both quadrilaterals and convex