3-5 Proving Lines Parallel

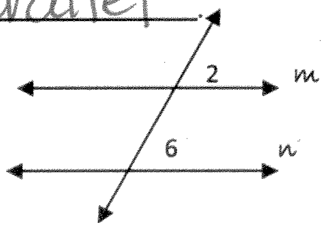
Objectives: Recognize angle conditions that occur with parallel lines.
Prove that two lines are parallel based on given angle relationships.

Master E

Converse of Corresponding Angles Postulate

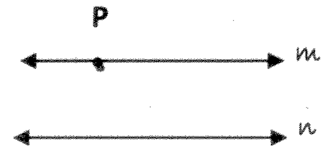
If two lines are cut by a transversal so that corresponding angles are ≅, then the 2 lines are parallel.

If: $\angle 2 \cong \angle 6$
Then: $m \parallel n$



Parallel Postulate

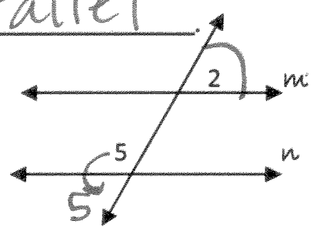
If given a line and a point not on that line, there exists exactly 1 line through the point that is parallel to the given line.



Alternate Exterior Angles Converse

If two lines are cut by a transversal so that a pair of alternate exterior angles is ≅, then the 2 lines are parallel.

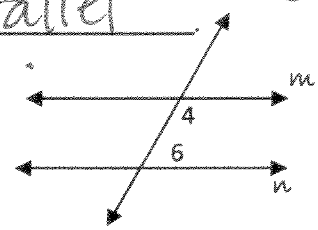
If: $\angle 2 \cong \angle 5$
Then: $m \parallel n$



Consecutive Interior Angles Converse

If two lines are cut by a transversal so that a pair of consecutive interior angles is supplementary, then the 2 lines are parallel.

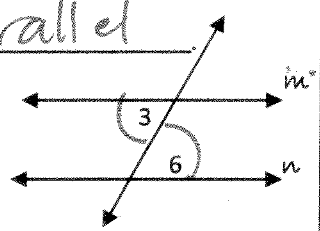
If: $m\angle 4 + m\angle 6 = 180$
Then: $m \parallel n$



Alternate Interior Angles Converse

If two lines are cut by a transversal so that a pair of alternate interior angles is ≅, then the 2 lines are parallel.

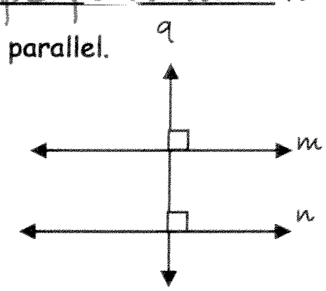
If: $\angle 3 \cong \angle 6$
Then: $m \parallel n$



Perpendicular Transversal Converse

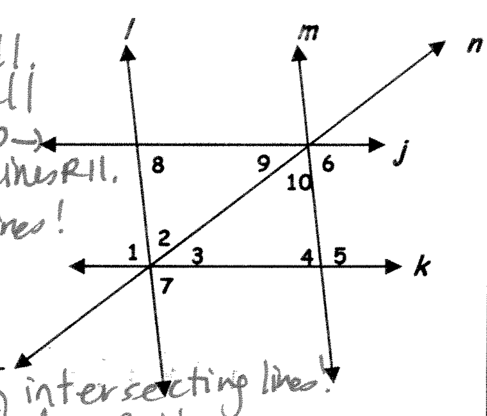
In a plane, if two lines are perpendicular to the same line, then they are parallel.

If: $q \perp m$ and $q \perp n$
Then: $m \parallel n$



Practice: **Would the following information make any two pairs of lines parallel? If yes, then which lines are parallel and why?**

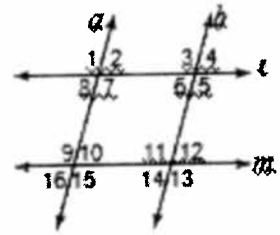
1. $\angle 1 \cong \angle 4$ l || m; If corr ∡s ≅ → lines R ||.
2. $\angle 6 \cong \angle 4$ j || k, If alt. int ∡s ≅ → lines R ||
3. $m\angle 2 + m\angle 3 = m\angle 5$ l || m, if corr ∡s R ≅ → lines R ||
4. $m\angle 2 + m\angle 3 + m\angle 8 = 180°$ j || k, If consec. int ∡s R supp →
5. $\angle 1 \cong \angle 8$ j || k, If alt int ∡s R ≅, → lines R || lines R ||.
6. $\angle 1 \cong \angle 7$ No vert. ∡s have nothing to do w/ || lines!
7. $\angle 2 \cong \angle 10$ l || m, If alt. int ∡s R ≅ → lines R ||.
8. $\angle 2 \cong \angle 9$ No, consec. int ∡s would be supp.
9. $m\angle 10 + m\angle 5 = 180°$ No. alt. int ∡s would be ≅
10. $m\angle 4 + m\angle 5 = 180°$ No, a linear pair is formed by intersecting lines!
11. $m\angle 1 = 90°$ and $m\angle 8 = 90°$ j || k, If alt int ∡s R ≅, → lines R ||



3-5 Skills Practice

Proving Lines Parallel

Given the following information, determine which lines, if any, are parallel. State the postulate or theorem that justifies your answer.



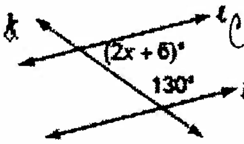
1. $\angle 3 \cong \angle 7$ a‖b
If alt int ∡s R ≅, → lines R ‖.

2. $\angle 9 \cong \angle 11$ a‖b
If corr ∡s R ≅, → lines R ‖.

3. $\angle 2 \cong \angle 16$ ℓ‖m
If alt. ext ∡s R ≅ → then lines R ‖.

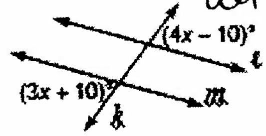
4. $m\angle 5 + m\angle 12 = 180$ ℓ‖m
If consec. int. ∡s R supp → lines R ‖

Find x so that $\ell \parallel m$. Show your work.

5. 

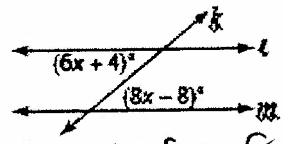
consec. int. (supp)

$2x+6+130=180$
$2x+136=180$
$2x=44$
$x=22$

6. 

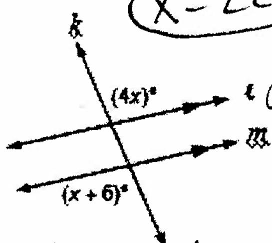
alt. ext (≅)

$3x+10=4x-10$
$10=x-10$
$20=x$

7. 

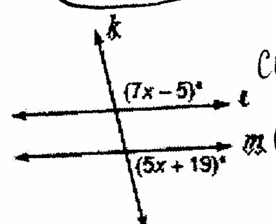
alt. int (≅)

$6x+4=8x-8$
$4=2x-8$
$12=2x$
$6=x$

8. 

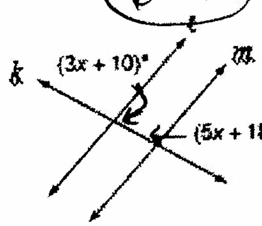
alt. ext (≅)

$4x=x+6$
$3x=6$
$x=2$

9. 

consec. ext. (supp)

$7x-5+5x+19=180$
$12x+14=180$
$12x=166$
$x=13.8$

10. 

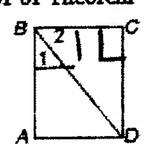
consec. int. (supp)

$3x+10+5x+18=180$
$8x+28=180$
$8x=152$
$x=19$

11. **PROOF** Provide a reason for each statement in the proof of Theorem 3.7.

Given: $\angle 1$ and $\angle 2$ are complementary.
$\overline{BC} \perp \overline{CD}$

Prove: $\overline{BA} \parallel \overline{CD}$



Proof:

Statements	Reasons
1. $\overline{BC} \perp \overline{CD}$	1. Given
2. $m\angle ABC = m\angle 1 + m\angle 2$	2. ∡ Add. Post.
3. $\angle 1$ and $\angle 2$ are complementary.	3. Given
4. $m\angle 1 + m\angle 2 = 90$	4. Def. comp. ∡s
5. $m\angle ABC = 90$	5. substitution POE (2 & 4)
6. ~~$\overline{BC} \perp \overline{CD}$~~ $\overline{BA} \perp \overline{BC}$	6. Def. ⊥ lines
7. ~~$\overline{BA} \perp \overline{CD}$~~ $\overline{BA} \parallel \overline{CD}$	7. In a plane, if 2 lines are ⊥ to the same line, then they are ‖.