Master Eq.

Day 02: 4-3 to 4-5 Proving Triangles Congruent

Congruent polygons have the following properties:

- Two geometric polygons are **congruent** if they have exactly the same shape and size.
- All the parts (sides & angles) of one polygon must be congruent to the corresponding parts of the other polygon.

Congruent Triangles (CPCTC-Corresponding Parts of Congruent Triangles are Congruent):

Definition of Congruent Triangles (CPCTC):

Two triangles are **congruent** if and only if their corresponding parts are congruent.

(3 corresponding sides & 3 corresponding angles).

Third Angles Theorem:

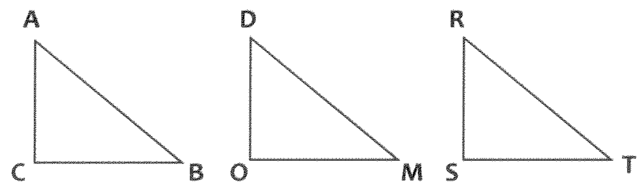
If two angles of one triangle are congruent to two angles of a second triangle, then the 3rd pair of angles are congruent.

Properties of Triangle Congruence (POTC):

Congruent Triangles are Reflexive:
$\Delta ABC \cong \Delta ABC$

Congruent Triangles are Symmetric:
If $\Delta ABC \cong \Delta DMO$, then $\Delta DMO \cong \Delta ABC$

Congruent Triangles are Transitive:
If $\Delta ABC \cong \Delta DMO$ and If $\Delta DMO \cong \Delta RTS$,
then If $\Delta ABC \cong \Delta RTS$



Congruence Statement: can be rearranged as long as the corresponding parts are in order.

If Δ **ABC** $\cong$ Δ**DMO**, then $\Delta BAC \cong \Delta MDO$, and $\Delta CAB \cong \Delta ODM$, and $\Delta BCA \cong \Delta MOD$

Corresponding Angles:

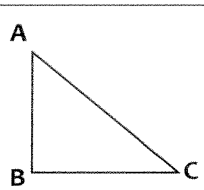
$\angle A \cong \angle D$, $\angle B \cong \angle M$, & $\angle C \cong \angle O$

Corresponding Sides:

$\overline{AB} \cong \overline{DM}$, $\overline{BC} \cong \overline{MO}$, & $\overline{AC} \cong \overline{DO}$

HINT: You don't need to look at the triangles to write these answers, just look at the congruence statement!

Go to https://www.explorelearning.com and search Proving Triangles Congruent.
Click **Launch Gizmo**. Click on each of the conditions and manipulate the 2 triangles to see what conditions are sufficient to prove the 2 triangles congruent. (A: $\cong$ corresponding angles and S: $\cong$ corresponding sides)



Adjacent example:

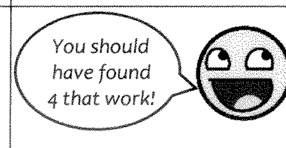
$\overline{BC}$ is adjacent to $\angle C$ or $\angle B$

Included examples:

$\overline{BC}$ is included between $\angle B$ & $\angle C$

$\angle C$ is included between $\overline{BC}$ & $\overline{AC}$

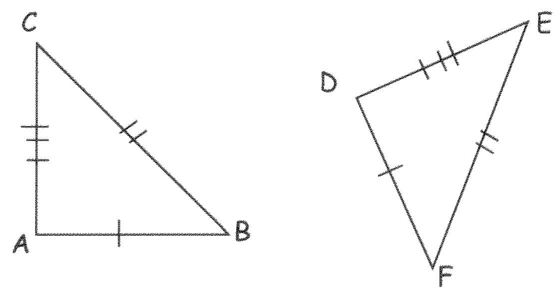
Are these conditions enough to prove the triangles congruent?
State YES or NO after your investigation as a class.

A (1 pair of $\cong \angle$s) NO	AA (2 pair of $\cong \angle$s) NO	AAA (3 pair of $\cong \angle$s) NO
S (1 pair of $\cong$ sides) NO	SS (2 pair of $\cong$ sides) NO	SSS (3 pairs of $\cong$ sides) YES
SA (side & adjacent angle $\cong$) NO	SSA (2 sides & a non-included angle $\cong$) NO	SAS (2 sides & the included angle $\cong$) YES
ASA (2 angles & the included side $\cong$) YES	AAS (2 angles & a non-included side $\cong$) YES	

Now that we know there are shortcuts to proving 2 triangles congruent without showing that all 6 pairs of corresponding parts congruent, let's look at these shortcuts in detail.

You must memorize these scenarios and notice that each one of them has 3 congruent marks.

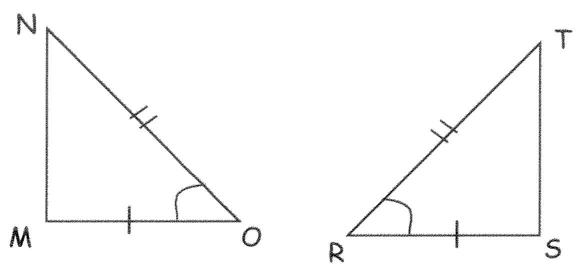
1. **SSS Postulate:** If 3 sides of one triangle are $\cong$ to 3 sides of a second triangle, then the two triangles are congruent.



If: Side $\overline{AC} \cong \overline{DE}$,
Side $\overline{AB} \cong \overline{DF}$, and
Side $\overline{CB} \cong \overline{EF}$
Then: $\Delta ABC \cong \Delta$ DFE

Definition: The angle formed by two adjacent sides of a polygon is called an included angle.

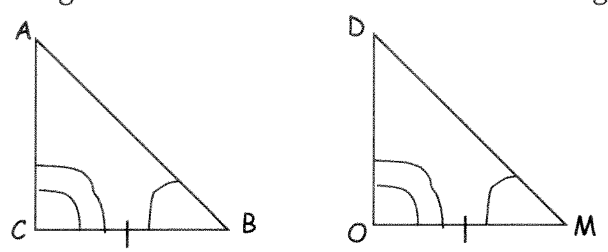
2. **SAS Postulate:** If 2 sides and the included angle of one triangle are congruent to the 2 sides and the included angle of a second triangle, then the two triangles are congruent.



If: Side $\overline{MO} \cong \overline{SR}$,
Angle $\angle O \cong \angle R$, and
Side $\overline{NO} \cong \overline{TR}$
Then: $\Delta MON \cong \Delta$ SRT by SAS

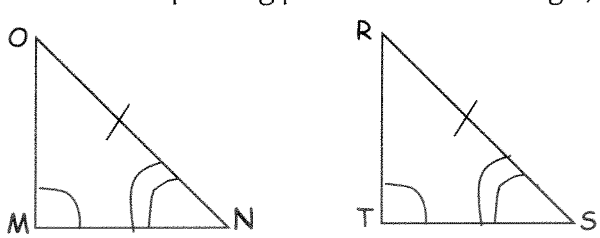
Definition: An included side is the side located between two consecutive angles of a polygon.

3. **ASA Postulate:** If 2 angles and the included side of one triangle are congruent to 2 angles and the included side of a second triangle, then the two triangles are congruent.



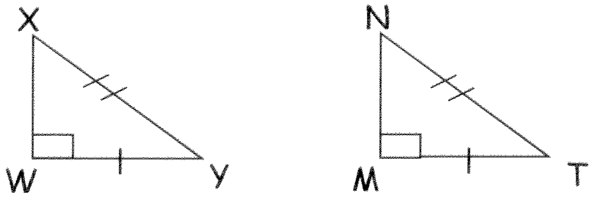
If: Angle $\angle C \cong \angle O$,
Side $\overline{CB} \cong \overline{OM}$, and
Angle $\angle B \cong \angle M$,
Then: $\Delta ABC \cong \Delta$ DMO by ASA

4. **AAS Postulate:** If 2 angles and an opposite or non-included side of one triangle are congruent to the corresponding parts of another triangle, then the two triangles are congruent.



If: Angle $\angle M \cong \angle T$,
Angle $\angle N \cong \angle S$, and
Side $\overline{ON} \cong \overline{RS}$
Then: $\Delta MON \cong \Delta$ TRS by AAS

5. **HL Theorem (only for 2 right triangles):** If the hypotenuse and a leg of one right triangle are congruent to the hypotenuse and corresponding leg of another right triangle, then the triangles are congruent.

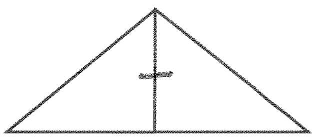


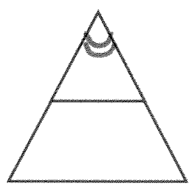
If: ΔXWY & ΔNMT are right triangles
Side $\overline{XY} \cong \overline{NT}$ and
Side $\overline{WY} \cong \overline{MT}$
Then: $\Delta XWY \cong \Delta$ NMT by HL

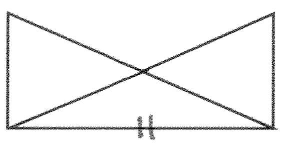
When you are given a set of triangles, even if the triangles appear to be congruent, you can only mark sides or angles congruent based on conditions in the picture that you have previously learned.

EXAMPLES WHEN NO INFORMATION IS GIVEN:

Reflexive Property of Equality: *If 2 triangles share the same side or angle, you can mark them congruent.*

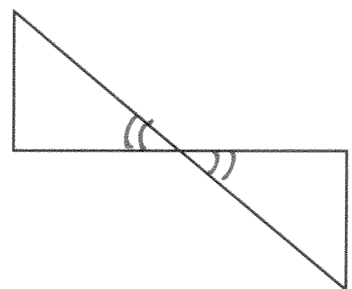






overlapping Δs

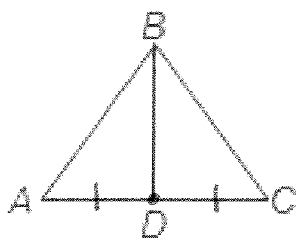
Vertical Angles: *they will always be congruent!*



EXAMPLES WHEN INFORMATION IS GIVEN:

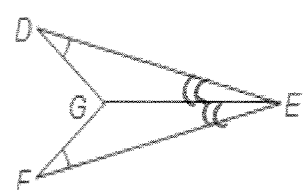
Midpoint: *will cut into 2 congruent segments!*

Given: D is the midpoint of $\overline{AC}$.

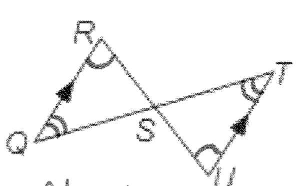


Angle Bisector: *will cut into 2 congruent angles!*

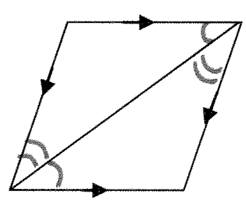
Given: $\overline{GE}$ bisects $\angle DEF$



Parallel Lines: *Parallel lines cut by a transversal will always form congruent corresponding and alternate interior angles.*



alt. int. ∡s



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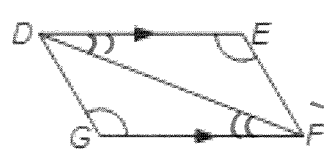
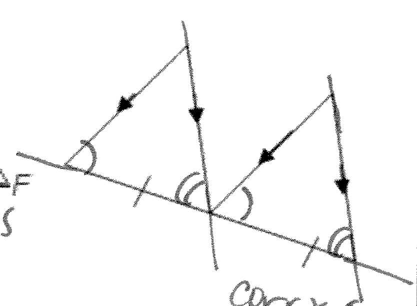


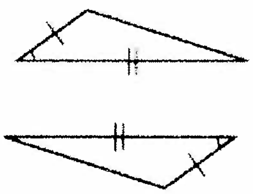
alt. int ∡s



corr ∡s

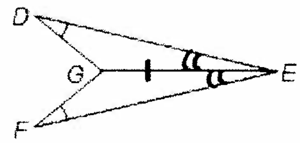
Decide if enough information is given to prove the triangles congruent. If there is enough information, state the reason (SSS, SAS, ASA, AAS, or HL). If there is not enough information, write NEI.

1. SAS

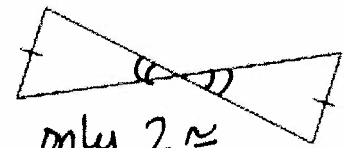


2. AAS

Given: $\overline{GE}$ bisects $\angle DEF$



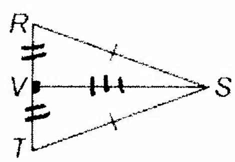
3. NEI



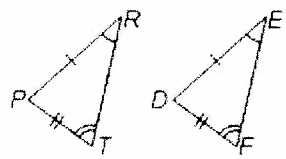
only 2 ≅ parts

4. SSS

Given: V bisects $\overline{RT}$

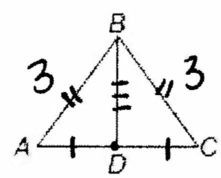


5. AAS

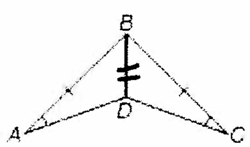


6. SSS

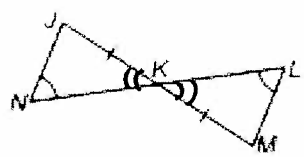
Given: D is the midpoint of $\overline{AC}$
AB = CB = 3 cm.



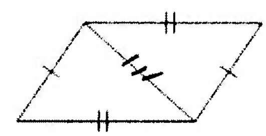
7. NEI



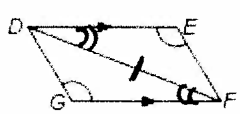
8. AAS



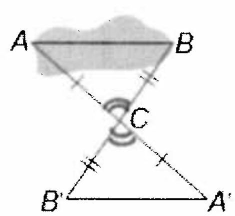
9. SSS



10. AAS

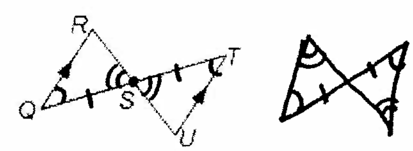


11. SAS

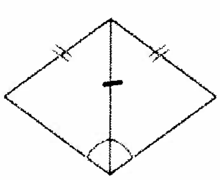


12. ASA or AAS

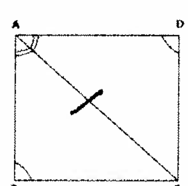
Given: S is the midpoint of $\overline{QT}$



13. NEI



14. AAS



15. HL