SOL AII.4   The student will solve, algebraically and graphically,
       a) absolute value equations and inequalities
       Graphing calculators will be used for solving and for confirming the algebraic solutions.

Hints and Notes

- An absolute value equation or inequality makes TWO statements.
- **Shading of Graphs**

$|absolute value| \leq number$	"AND" sentence (Less than or equal to)	Shaded between 2 numbers and closed circles
$|absolute value| < number$	"AND" sentence (Less than only)	Shaded between 2 numbers and open circles
$|absolute value| \geq number$	"OR" sentence (Greater than or equal to)	Shaded to the left and right, closed circles
$|absolute value| > number$	"OR" sentence (Greater than only)	Shaded to the left and right, open circles

PRACTICE AII.4a

1. Which of the following represents the solution to $|x| = 7$? $X = 7 \quad -X = 7 \quad \therefore \quad X = -7$

   A $x = 7$
   B $x = 0$
   C $x = -7$
   D $x = 7$ or $x = -7$

2. Which inequality describes the solution set graphed? $X < 0 \quad X > \frac{1}{3}$
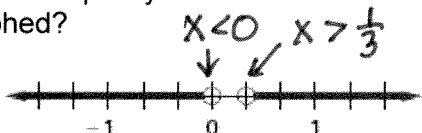
   A $-3|4x - 1| + 7 < 2$
   B $-3|4x - 1| + 7 > 2$
   C $-2|6x - 1| + 5 < 3$
   D $-2|6x - 1| + 5 > 3$
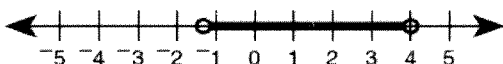
3. What is the solution to $|2x - 3| - 1 < 3$?

   A $\dfrac{-1}{2} < x < \dfrac{7}{2}$
   B $\dfrac{-7}{2} < x < \dfrac{7}{2}$
   C $x > -\dfrac{1}{2}$ or $x < \dfrac{7}{2}$
   D $x = -\dfrac{1}{2}$ or $x = \dfrac{7}{2}$

$|2x - 3| < 4$ AND
$2x - 3 < 4 \qquad 2x - 3 > 4$
$2x < 7 \qquad 2x > -1$
$x < \frac{7}{2} \qquad x > \frac{1}{2}$

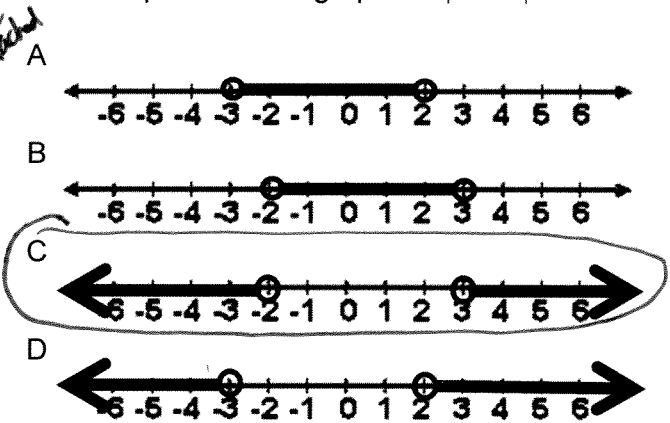**4.** Which inequality describes the solution set graphed?

   A $|3x - 4| > 8$
   B $|3x - 4| < 8$
   C $|2x - 3| > 5$
   D $|2x - 3| \leq 5$

5. Which is the solution set for $|2x + 5| = 7$?

   A $\{x \mid x = -6 \ \text{ or } \ x = 1\}$
   B $\{x \mid x = -1 \ \text{ or } \ x = 1\}$
   C $\{x \mid x = -6\}$
   D $\{x \mid x = 1\}$

$2x + 5 = 7 \qquad 2x + 5 = -7$
$2x = 2 \qquad\quad 2x = -12$
$x = 1 \qquad\qquad x = -6$

6. Which represents the graph of $2|2x - 1| > 10$?

   A

   B

   C

   D

7. Which of the following inequalities best represents

   A $|x - 1| < 3$ AND $\begin{array}{ll} x - 1 < 3 & x - 1 > -3 \\ x < 4 & x > -2 \end{array}$
   B $|x - 7| < 4$ $\begin{array}{ll} x - 7 < 4 & x - 7 > -4 \\ x < 11 & x > 3 \end{array}$
   C $|x + 3| < 7$ $\begin{array}{ll} x + 3 < 7 & x + 3 > -7 \\ x < 4 & x > -11 \end{array}$
   D $|x + 3| < 3$ $\begin{array}{ll} x + 3 < 3 & x + 3 > -3 \\ x < 0 & x > -6 \end{array}$

2.

A. $-3|4x-1|+7<2$
$-3|4x-1|<-5$
$|4x-1|>\frac{5}{3}$
∧

$4x-1>\frac{5}{3}$ $-(4x-1)>\frac{5}{3}$
$4x>\frac{8}{3}$ $4x-1<-\frac{5}{3}$
$x>\frac{8}{3}\cdot\frac{1}{4}$ $4x<-\frac{2}{3}$
$x>\frac{2}{3}$ $x<-\frac{2}{3}\cdot\frac{1}{6}$ ... $x<-\frac{1}{6}$

B. $-3|4x-1|+7>2$
$-3|4x-1|>-5$
$|4x-1|<\frac{5}{3}$
∧

$4x-1<\frac{5}{3}$ $4x-1>-\frac{5}{3}$
$x<\frac{2}{3}$ $x>-\frac{5}{3}\div\frac{1}{6}$ ... $x>-\frac{1}{6}$

Ⓒ $-2|6x-1|+5<3$
$-2|6x-1|<-2$
$|6x-1|>1$
∧

$6x-1>1$ $6x-1<-1$
$6x>2$ $6x<0$
$x>\frac{1}{3}$ $x<0$

D. $-2|6x-1|+5>3$
$-2|6x-1|>-2$
$|6x-1|<1$
∧

$6x-1<1$ $6x-1>-1$
$6x<2$ $6x>0$
$x<\frac{1}{3}$ $x>0$

4.

A. $|3x-4|\geq 8$ "OR"
∧
$3x-4\geq 8$ $3x-4\leq -8$
$3x\geq 12$ $3x\leq 4$
$x\geq 4$ $x\leq \frac{4}{3}$

B. $|3x-4|<8$ "AND"
∧
$3x-4<8$ $3x-4>-8$
$3x<12$ $3x>-4$
$x<4$ $x>\frac{4}{3}$

C. $|2x-3|>5$ "OR"
∧
$2x-3>5$ $2x-3<-5$
$2x>8$ $2x<-2$
$x>4$ $x<-1$

D. $|2x-3|\leq 5$ "AND"
∧
$2x-3\leq 5$ $2x-3\geq -5$
$2x\leq 8$ $2x\geq -2$
$x\leq 4$ $x\geq -1$

6.

$2|2x-1|>10$
$|2x-1|>5$ "OR"
∧

$2x-1>5$ $2x-1<-5$
$2x>6$ $2x<-4$
$x>3$ $x<-2$

SOL AII.4   The student will solve, algebraically and graphically,
  b) quadratic equations over the set of complex numbers
  Graphing calculators will be used for solving and for confirming the algebraic solutions.

HINTS and NOTES

- Methods for solving a quadratic equation:
- From $ax^2 + bx + c = 0$ form, GRAPH, then find the x-intercepts/zeros.
- From $ax^2 + bx + c = 0$ form, FACTOR, then set each factor equal to zero and solve for x.
- Isolate a squared term or squared binomial on one side, then take the SQUARE ROOT of both sides and solve for x.
- From $ax^2 + bx + c = 0$ form, use QUADRATIC FORMULA: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
- *Work backward* by substituting the given answer choices into a graphics calculator to find the solution(s) that satisfy the equation.

PRACTICE AII.4b

1. Which is the solution set for $x^2 + 6x - 16 = 0$?

  A $\{0, 4\}$

  B $\{-8, 2\}$

  C $\{-3, 5\}$

  D $\{-2, 8\}$

Handwritten: $(x+8)(x-2)=0$ $-8 \quad 2$

2. Which is the solution set for $x^2 - 6x = 8$?

  A $\{2 \pm 2i\}$

  B $\{2 \pm 2\sqrt{3}\}$

  C $\{4, 2\}$

  D $\{-4, 2\}$

Handwritten: $x^2 - 6x + 8 = 0$ $(x-4)(x-2) = 0$ $x = 4, \ x = 2$

3. Which is the solution set for $2x^2 + 2x + 1 = 0$?

  A $\left\{\pm\dfrac{1}{2}\right\}$

  B $\left\{\dfrac{-1}{2} \pm \dfrac{1}{2}i\right\}$

  C $\left\{\dfrac{-1}{2} \pm i\right\}$

  D $\{-1 \pm i\}$

Handwritten: $\dfrac{-2 \pm \sqrt{4 - 4(2)}}{4}$ $\dfrac{-2 \pm \sqrt{-4}}{4}$ $\dfrac{-2 \pm 2i}{4} = \dfrac{-1 \pm i}{2}$ $-\dfrac{1}{2} \pm \dfrac{i}{2}$

4. What are the solutions to $(y+3)^2 - 81 = 0$?

  A $y = -12$ or $y = -6$

  B $y = -12$ or $y = 6$

  C $y = 12$ or $y = -6$

  D $y = 12$ or $y = 6$

Handwritten: $(y+3)^2 = 81$ $y+3 = \pm 9$ $y = -3 + 9 = 6$ $y = -3 - 9 = -12$

5. Which are the solutions for $x^2 - 3x - 4 = 0$?

  A $x = 1$ or $x = -4$

  B $x = -1$ or $x = 4$

  C $x = \dfrac{3 \pm i\sqrt{7}}{2}$

  D $x = \dfrac{3 \pm \sqrt{7}}{2}$

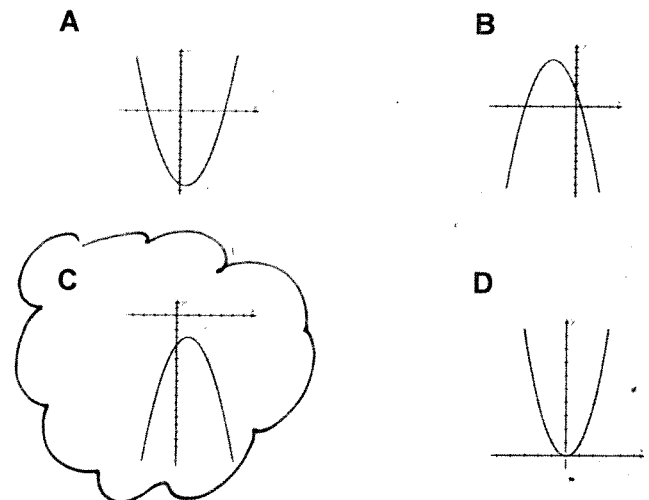*Handwritten:* $(x-4)(x+1) = 0$ $4, -1$

6. Which are the solutions for $25x^2 - 10x + 1 = 0$?

  A $\{5\}$

  B $\left\{\dfrac{1}{5}\right\}$

  C $\{-5, 5\}$

  D $\left\{-\dfrac{1}{5}, \dfrac{1}{5}\right\}$

Handwritten: $(5x-1)(5x-1) = 0$ $x = \dfrac{1}{5} \quad x = \dfrac{1}{5}$

7. Which graph represents a quadratic equation with no real solutions?

A

B

C

D

SOL AII.4 **The student will solve, algebraically and graphically,**
 c) equations containing rational algebraic expressions
 Graphing calculators will be used for solving and for confirming the algebraic solutions.

HINTS and NOTES
To solve a rational equation:
- Multiply both sides of the equation (**_every_** term) by the Least Common Denominator and "clear the denominators".
- When the equation is a proportion (one fraction = one fraction), cross-multiply the expressions across =.
- For a multiple choice problem, work backward by substituting each given value into the equation to determine which one satisfies the equation.
- Graph each side of the equation in two Y= on a graphics calculator and find the point or points of intersection.

PRACTICE AII.4c

1. Which is the solution for $\dfrac{x}{2x+1} = \dfrac{4}{3}$ **?**

A $x = \dfrac{-1}{5}$

B $x = 5$

C $x = \dfrac{-4}{5}$ (circled)

D $x = \dfrac{-5}{4}$

handwritten:
$3x = 8x + 4$
$-5x = 4$
$x = -\dfrac{4}{5}$

2. Which value of q is the solution for the equation $\dfrac{7q-9}{6} = \dfrac{6q+2}{4}$ **?**

A $q = \dfrac{-11}{8}$

B $q = -6$ (circled)

C $q = \dfrac{31}{9}$

D $q = 48$

handwritten:
$4(7q-9) = 6(6q+2)$
$28q - 36 = 36q + 12$
$-48 = 8q$
$q = -6$

3. Which is the solution for $\dfrac{3x^2 - 2}{x} = \dfrac{6x - 2}{x}$ **?**

A 6

B 2 (circled)

C $\sqrt{2}$

D 0

handwritten:
$3x^2 - 2 = 6x - 2$
$3x^2 - 6x = 0$
$3x(x-2) = 0$
$0 \quad 2 \quad x \neq 0$

3. Which value of y is the solution for the equation $\dfrac{4y-30}{3} + \dfrac{6y+8}{2} = 9$ **?**

A $y = \dfrac{28}{5}$

B $y = \dfrac{45}{13}$ (circled)

C $y = \dfrac{8}{5}$

D $y = \dfrac{23}{24}$

handwritten:
$\dfrac{2(4y-30)+3(6y+8)}{6} = 9$
$\dfrac{8y-60+18y+24}{6} = 9$
$\dfrac{26y-36}{6} = 9 \qquad \rightarrow y = \dfrac{98}{26}$
$26y-36 = 54 \qquad = \dfrac{45}{13}$
$26y = 98$

TEI (Technology Enhanced Item): Free-Response - For free-response questions, type your answer in the box. Be sure your answer is in appropriate form - simplest fraction, decimal, etc.

4. Solve the following equation for x. Place your answer in the box.

$$x + \dfrac{25}{x} = 10$$

handwritten:
$\dfrac{x^2}{x} + \dfrac{25}{x} = 10 \rightarrow x^2 - 10x + 25 = 0$
$(x-5)^2 = 0$
$\dfrac{x^2 + 25}{x} = 10$
$x^2 + 25 = 10x$

$\boxed{5}$

SOL AII.4 The student will solve, algebraically and graphically,
　　　d) equations containing radical expressions
　　　　Graphing calculators will be used for solving and for confirming the algebraic solutions.

HINTS and NOTES
To solve radical equation:
- Isolate the radical on one side of the equation, then raise both sides of the equation to the power equal to the index (If square roots, square both sides; if cube roots, cube both sides, etc.), and solve for x.
- If the equation contains two radicals, manipulate the equation so the one radical is on each side of the equation, then raise both sides of the equation to the power equal to the index, and solve for x.
- For a multiple choice problem, work backward by substituting each given value into the equation to determine which one satisfies the equation.
- Graph each side of the equation in two Y= on a graphics calculator and find the point or points of intersection.

PRACTICE AII.4d

1. Which is the solution set for $\sqrt{x-4}=5$?

　A $\{21\}$
　B $\{25\}$
　C $\{29\}$
　D $\{33\}$

$x-4=25$
$x=29$

2. Which is the solution set for $\frac{1}{4}\sqrt{9+x}=1$?

　A $\{-7,7\}$
　B $\{-5,5\}$
　C $\{7\}$
　D $\{5\}$

$\sqrt{9+x}=4$
$9+x=16$
$x=7$

3. What is the solution set for $\sqrt{3y}+4=5$?

　A $\{y\,|\,y=3\}$
　B $\{y\,|\,y=1\}$
　C $\{y\,|\,y=\frac{1}{3}\}$
　D $\{y\,|\,y=\frac{1}{9}\}$

$\sqrt{3y}=1$
$3y=1$
$y=\frac{1}{3}$

4. Which is the solution for $\sqrt{x+16}=3\sqrt{x}$?

　A $\{x\,|\,x=\frac{1}{2}\}$
　B $\{x\,|\,x=\frac{8}{5}\}$
　C $\{x\,|\,x=2\}$
　D $\{x\,|\,x=5\}$

$x+16=9x$
$16=8x$
$2=x$

5. Which is the solution to $\sqrt[3]{x-3}+3=5$?

　A $x=2$
　B $x=3$
　C $x=7$
　D $x=11$

$\sqrt[3]{x-3}=2$
$x-3=8$
$x=11$

6. The length, s, (in feet) of the skid mark left by an automobile traveling at r miles per hour can be approximated by the relation $r=2\sqrt{5s}$. If a car is going 80 miles per hour when the brakes are applied, about how many feet long would the skid mark be?

　A $320\,ft$
　B $410\,ft$
　C $640\,ft$
　D $1280\,ft$

$r=2\sqrt{5s}$
$80=2\sqrt{5s}$
$40=\sqrt{5s}$
$1600=5s$
$320=s$

SOL AII.5   The student will solve nonlinear systems of equations, including linear-quadratic and quadratic-quadratic, algebraically and graphically. Graphing calculators will be used as a tool to visualize graphs and predict the number of solutions

HINTS and NOTES
- The solution for a system is at the point or points of intersection of the equations.
- The coordinates of the solution(s) for a system of equations will satisfy both equations in the system.
- If given a graph, locate the point(s) of intersection and identify the coordinates.
- If given equations, solve by **graphing** or solve algebraically using **substitution** or **elimination**.
- To solve by graphing on a graphics calculator, isolate y on one side of each equation, put each equation into Y= , and find the point of intersection (2nd CALC > Intersect).
- If the curves do not intersect at all, then there is no solution for the system.

PRACTICE AII.5

1. **Which is the solution set to the following system of equations?**

$$\begin{cases} y + 2x = 2 \\ x^2 + 3y = 22 \end{cases}$$

(handwritten: $y = -2x + 2$, $x^2 + 3(-2x+2) = 22$, $x^2 - 6x + 6 = 22$, $x^2 - 6x - 16 = 0$, $(x-8)(x+2)$, $8 \quad -2$, $x=8 \quad y = -2(8)+2 = -14$, $x=2 \quad y = -2(2)+2 = 6$)

A $\{(-8,18)$ and $(2,-2)\}$
B $\{(-8,2)$ and $(18,-2)\}$
C $\{(-2,2)$ and $(18,-8)\}$
D $\{(8,-14)$ and $(-2,6)\}$

2. **Which set of ordered pairs is the solution for the system of equations?**

$$\begin{cases} y = x^2 - 2x - 1 \\ y = -x^2 + 4x - 1 \end{cases}$$

(handwritten: $x^2 - 2x - 1 = -x^2 + 4x - 1$, $2x^2 - 6x = 0$, $2x(x-3) = 0$, $0, 3 = x$, $x = 0 \quad y = -1$, $x = 3 \quad y = 9 - 6 - 1 = 2$)

A $\{(0,3)$ and $(-1,2)\}$
B $\{(0,-1)$ and $(3,2)\}$
C $\{(0,-1)$ and $(6,23)\}$
D $\{(3,2)$ and $(6,-11)\}$

3. **Which of the following is the solution for the system of equations?**

$$\begin{cases} 2y = x^2 - 6x - 9 \\ 2y = -x^2 + 2x + 1 \end{cases}$$

(handwritten: $x^2 - 6x - 9 = -x^2 + 2x + 1$, $2x^2 - 8x - 10 = 0$, $2(x^2 - 4x - 5) = 0$, $2(x-5)(x+1) = 0$, $x = 5, -1$)

A $\{(5,-7)$ and $(-1,-1)\}$
B $\{(1,1)$ and $(-5,23)\}$
C $\{(1,-7)$ and $(-5,23)\}$
D $\{(2,\frac{1}{2})\}$

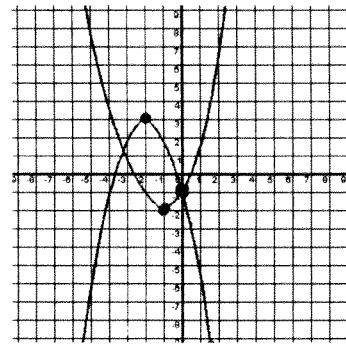*(handwritten at bottom: $x=5 \quad 2y = 25 - 30 - 9 = -14 \quad y = -7$, $x = -1 \quad 2y = 1 + 6 - 9 = -2 \quad y = -1$)*

4. **Which is most likely the solution set for the system of equations?**

(handwritten: $(0,-1)$)

A $\{(-1.2,-3.2), (-1,0)\}$
B $\{(-1,-2), (-2,3)\}$
C $\{(-2,3), (1,-2)\}$
D $\{(-3.2,1.2), (0,-1)\}$

5. **Which is most likely the solution set for the system represented in the graph?**
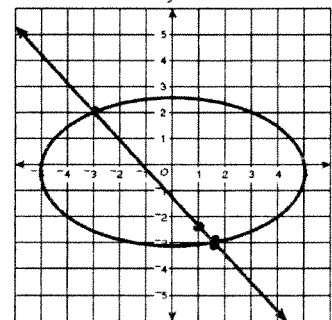
(handwritten: $(-3,2)$, $(1.5,-3)$)

A $\{(1.5,2.5)$ and $(3,2)\}$
B $\{(-2.5,1.5)$ and $(2,-3)\}$
C $\{(-2,-3)$ and $(2.5,-1.5)\}$
D $\{(-3,2)$ and $(1.5,-3)\}$