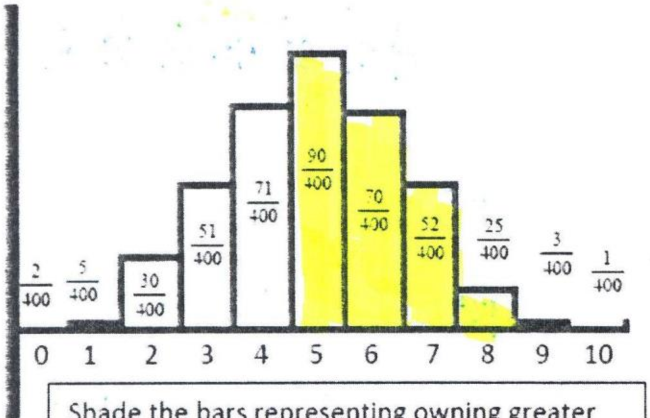
Normal Distribution: Finding Probabilities

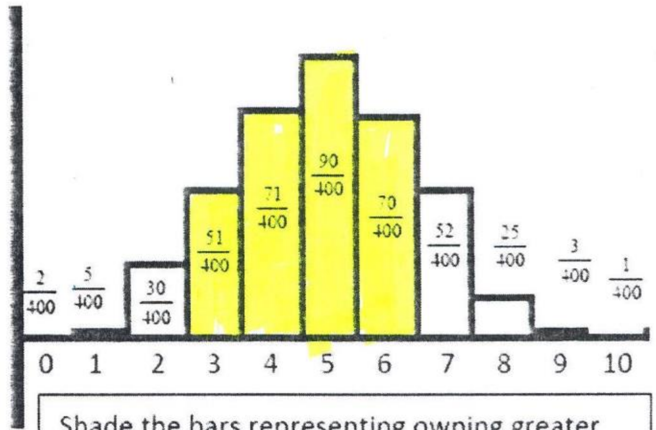
Shading the Probability

1. The histograms below reflect the number of pets veterinarians own. The value associated with each bar represents the fraction of veterinarians with that many pets.



Shade the bars representing owning greater than or equal to 5 pets. What fraction has 5 or more pets?

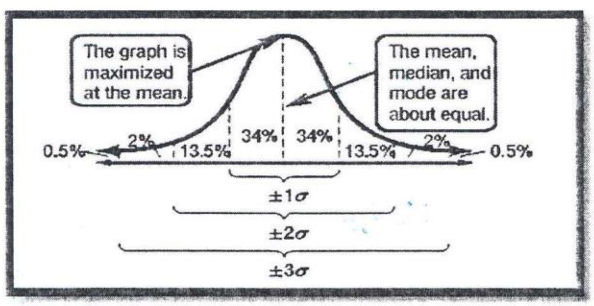
$\frac{241}{400}$



Shade the bars representing owning greater than 2 but less than 7 pets. What fraction of owners falls into this group?

$\frac{282}{400} = \frac{141}{200}$

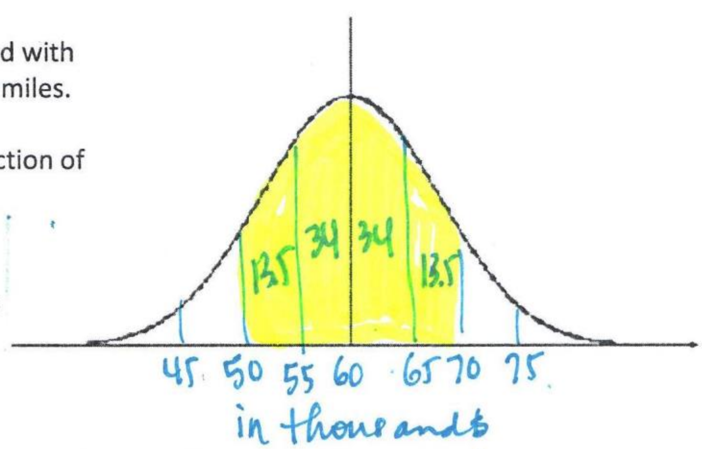
Finding the Area under a Normal Curve for values with integer z-scores



2-3: Represent the following using a normal distribution graph. Show three standard deviations to the left and right of the mean.

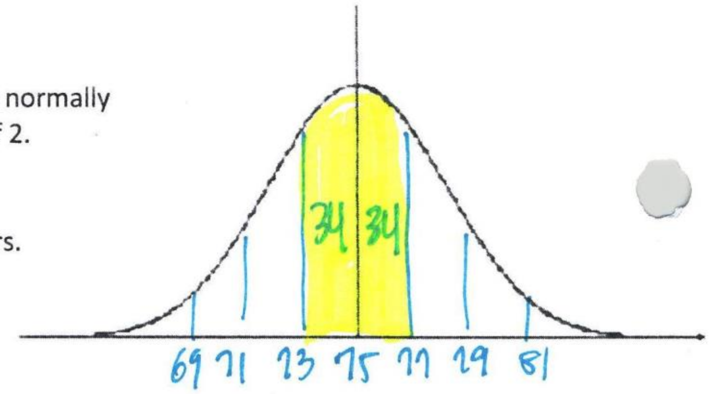
2. The length of wear on Spinning Tires is normally distributed with a mean of 60,000 miles and a standard deviation of 5,000 miles.

a. Shade the region under the curve that represents the fraction of tires that last between 50,000 miles and 70,000 miles.



b. According to the Empirical Rule, what percentage of tires does this represent? 95%

3. The number of crackers in a box of Crackerbox Crackers is normally distributed with a mean of 75 and a standard deviation of 2.

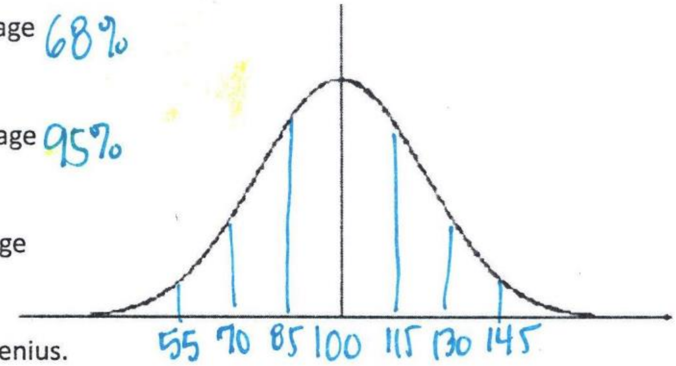


a. Shade the region under the curve that represents the probability that a box has between 73 and 77 crackers.

b. According to the Empirical Rule, what is this probability?

68%

4. Suppose that IQ Scores have a normal distribution with a mean of 100 and a standard deviation of 15.



a. According to this data and the Empirical Rule, what percentage of people have an IQ score between 85 and 115? 68%

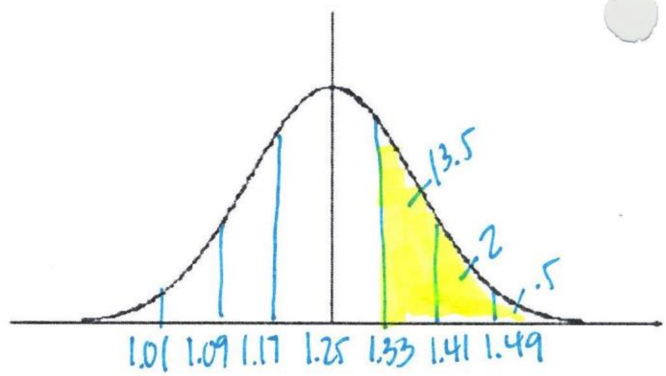
b. According to this data and the Empirical Rule, what percentage of people have an IQ score between 70 and 130? 95%

c. According to this data and the Empirical Rule, what percentage of people have an IQ score above 130? 2.5%

d. A person with an IQ score greater than 145 is considered a genius. Does the Empirical Rule support this statement? Explain.

the top .5% of the population would definitely indicate that someone is a genius.

5. The price of a certain kind of chocolate bar is normally distributed with a mean of $1.25 and a standard deviation of $0.08.

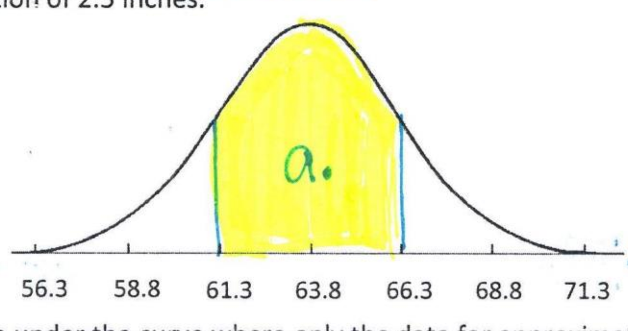


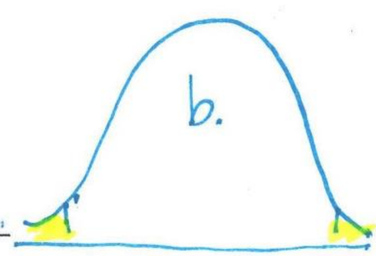
a. According to this data and the Empirical Rule, what percent of the chocolate bars cost less than $1.25? 50%

b. According to this data and the Empirical Rule, what percent of the chocolate bars cost more than $1.25? 50%

c. According to this data and the Empirical Rule, what percent of the chocolate bars cost more than $1.33? 16%

6. The graph below summarizes the heights of 40, 000 women. The data is normally distributed with a mean of 63.8 inches and a standard deviation of 2.5 inches.





a. Identify and shade the region under the curve where only the data for approximately 27200 women are located.

27,200 = x·40,000 = 68%

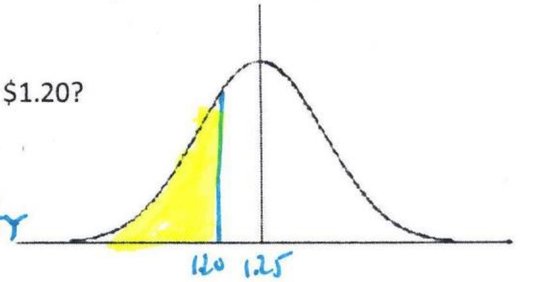
b. Identify and shade the regions under the curve where only the data for approximately 32600 women are located.

32,600= x · 40,000 = 1.23%.

Calculating the Area under a Normal Curve for values with non-integer z-scores

7. The price of a certain kind of chocolate bar is normally distributed with a mean of \$1.25 and a standard deviation of \$0.08.

a. What is the probability that the chocolate bar has a price that is less than \$1.20?



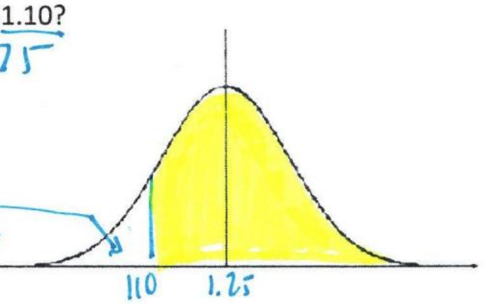
$\boxed{-.63}$

z-score = $\boxed{-.625}$ area = 0.2643 ⇒ 26.43% PROBABILITY

$\frac{1.20-1.25}{.08}$ round up!

b. What is the probability that the chocolate bar has a price that is more than \$1.10?

$\frac{1.10-1.25}{.08} = -1.875$



z-score of \$1.10 = -1.88

area bound by \$1.10 = .0301 WHITE

area more than \$1.10 = .9699 1−.0301 YELLOW

PROB: 96.99%

c. What is the probability that the chocolate bar has a price between \$1.05 and \$1.45?

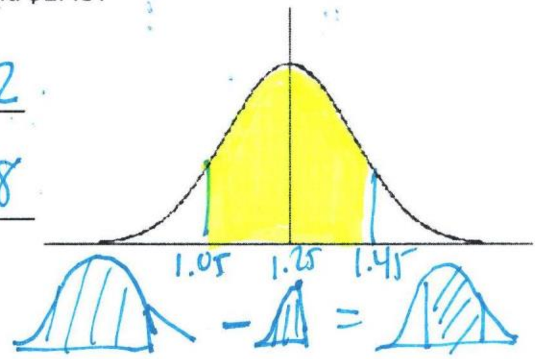
$\frac{1.05-1.25}{.08}$

left z-score of \$1.05 = -2.5 area bound by \$1.05 = .0062

$\frac{1.45-1.25}{.08}$

right z-score of \$1.45 = 2.5 area bound by \$1.45 = .9938

.9938−.0062



area between \$1.05 and \$1.45 = .9876

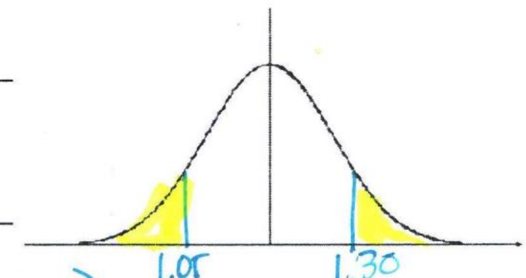
PROB: 98.76%

d. What is the probability that the chocolate bar is more than \$1.30 or less than \$1.05?

z-score of \$1.05 = -2.5 area bound by \$1.05 = .0062

$\frac{1.30-1.25}{.08} = .625$

z-score of \$1.30 = .63 area bound by \$1.30 = .7357



total shaded area = .2705 .0062+(1−.7357)

PROB: 27.05%

e. Out of 200 candy bars, how many would we expect to have a price of less than \$1.20? 52.86 (from a)

How many of the 200 would we expect to have a price between \$1.05 and \$1.45? 197.52 (from c)

z-score = −.63 area = .2643 .2643(200)

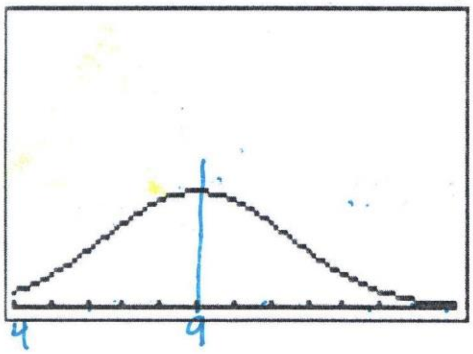
.9876(200)

3. Each of the normal curves below is graphed in the viewing window given at right on a graphing calculator. One of each of the graphs has a mean of either 7, 8, 9, or 10, and one of each of the graphs has a standard deviation of 1, 1.5, 2, or 2.5.

Use the above information to determine the mean and standard deviation of each graph. Then check your answers by graphing each in the y= screen and comparing the viewing screen to the given graph.

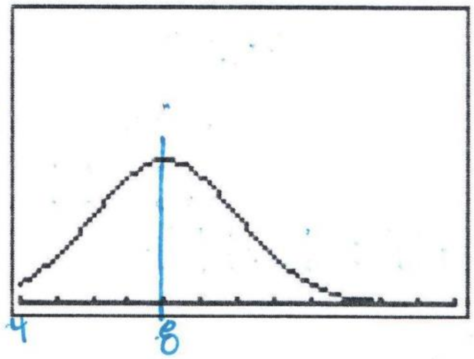
```
WINDOW
 Xmin=4
 Xmax=16
 Xscl=1
 Ymin=0
 Ymax=.4
 Yscl=1
 Xres=1
```

a.



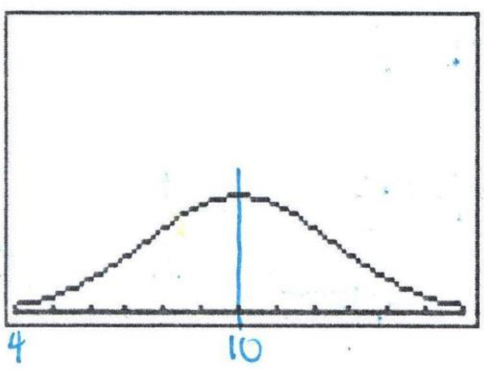
μ = 9 σ = 2

c.



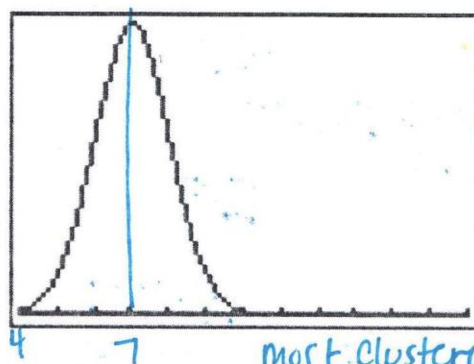
μ = 8 σ = 1.5

b.



μ = 10 σ = 2.5

d.



most clustered

μ = 7 σ = 1

Remember: the greater the σ, the more spread out the data!