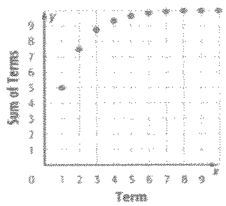
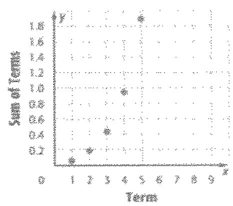
11-4 Infinite Geometric Series

Infinite Geometric Series A geometric series that does not end is called an infinite geometric series. Some infinite geometric series have sums, but others do not because the partial sums increase without approaching a limiting value.

Key Concept Convergent and Divergent Series

Convergent Series		Divergent Series	
Words	The sum approaches a finite value.	Words	The sum does not approach a finite value.
Ratio	$\lvert r \rvert < 1$	Ratio	$\lvert r \rvert \geq 1$
Example	$5 + 2.5 + 1.25 + \ldots$	Example	$\frac{1}{16} + \frac{1}{8} + \frac{1}{4} + \ldots$

Sum of an Infinite Geometric Series	$S = \frac{a_1}{1-r}$ for $-1 < r < 1$. If $\lvert r \rvert \geq 1$, the infinite geometric series does not have a sum.

$r = \frac{a_n}{a_{n-1}}$

Examples: Find the sum of each infinite series, if it exists.

a. $75 + 15 + 3 + \ldots$

First, find the value of r to determine if the sum exists. $a_1 = 75$ and $a_2 = 15$, so $r = \frac{15}{75}$ or $\frac{1}{5}$. Since $\left|\frac{1}{5}\right| < 1$, the sum exists. Now use the formula for the sum of an infinite geometric series.

$S = \frac{a_1}{1-r}$ Sum formula

$= \frac{75}{1-\frac{1}{5}}$ $a_1 = 75, r = \frac{1}{5}$

$= \frac{75}{\frac{4}{5}}$ or 93.75 Simplify.

The sum of the series is 93.75.

b. $\sum_{n=1}^{\infty} 48\left(-\frac{1}{3}\right)^{n-1}$

In this infinite geometric series, $a_1 = 48$ and $r = -\frac{1}{3}$.

$S = \frac{a_1}{1-r}$ Sum formula

$= \frac{48}{1-\left(-\frac{1}{3}\right)}$ $a_1 = 48, r = - -1$

$= \frac{48}{\frac{4}{3}}$ or 36 Simplify.

Thus $\sum_{n=1}^{\infty} 48\left(-\frac{1}{3}\right)^{n-1} = 36$.

Exercises: Determine whether each infinite geometric series is convergent or divergent. Find the sum of each infinite series, if it exists. C D

1. $a_1 = -7, r = \frac{5}{8}$ <1

C $\frac{-7}{\left(1-\frac{5}{8}\right)} = \frac{-56}{3}$

2. $1 + \frac{5}{4} + \frac{25}{16} + \cdots$

D $r = \frac{5}{4} > 1$

No sum exists!

3. $a_1 = 4, r = \frac{1}{2}$

C $\frac{4}{\left(1-\frac{1}{2}\right)} = 8$

4. $\frac{2}{9} + \frac{5}{27} + \frac{25}{162} + \cdots$ $r = \frac{5}{27} \div \frac{2}{9} = \frac{5}{27} \cdot \frac{9}{2} = \frac{5}{6}$

C $\frac{\frac{2}{9}}{1-\frac{5}{6}} = \frac{4}{3}$

5. $18 - 9 + 4\frac{1}{2} - 2\frac{1}{4} + \cdots$ $r = -\frac{1}{2}$

C $\frac{18}{1-\left(-\frac{1}{2}\right)} = \frac{18}{1.5} = 12$

6. $6 - 12 + 24 - 48 + \cdots$

D $r = -2$ $\lvert r \rvert > 1$

No sum exists!

7. $\sum_{n=1}^{\infty} 50\left(\frac{4}{5}\right)^{n-1}$ r

C $\frac{50}{1-\frac{4}{5}} = 250$

8. $\sum_{k=1}^{\infty} 22\left(-\frac{1}{2}\right)^{k-1}$

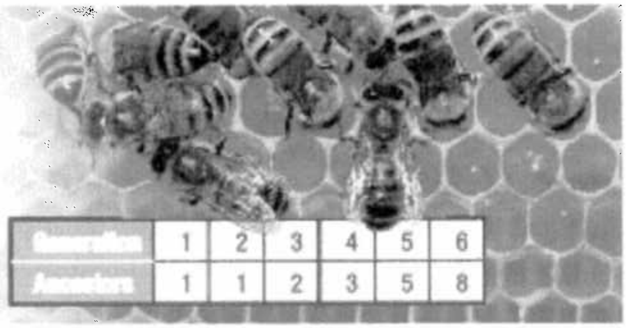
C $\frac{22}{1-\left(-\frac{1}{2}\right)} = \frac{44}{3}$

9. $\sum_{s=1}^{\infty} 24\left(\frac{7}{12}\right)^{s-1}$

C $\frac{24}{1-\frac{7}{12}} = \frac{288}{5}$

11-5 Recursively Defined Sequences

The female honeybee is produced after the queen mates with a male, so the female has two parents, a male and a female. The male honeybee, however, is produced by the queen's unfertilized eggs and thus has only one parent, a female. The family tree for the honeybee follows a special sequence.



Notice that every term in the list of ancestors is the sum of the previous two terms. This special sequence is called the Fibonacci sequence, and it is found in many places in nature. The Fibonacci sequence is an example of a recursive sequence. In a recursive sequence, each term is determined by one or more of the previous terms.

The formulas we have used for sequences thus far have been explicit formulas. An explicit formula gives a_n as a function of n, such as $a_n = 3n + 1$. The formula that describes the Fibonacci sequence, $a_n = a_{n-2} + a_{n-1}$, is a recursive formula, which means that every term will be determined by one or more of the previous terms. An initial term must be given in a recursive formula.

Key Concept **Recursive Formulas for Sequences**

Arithmetic Sequence $a_n = a_{n-1} + d$, where d is the common difference

Geometric Sequence $a_n = r \cdot a_{n-1}$, where r is the common ratio

- **Recursion:** each step in a sequence depends on the step before it.
- **Recursive Formula:** a formula that defines a sequence. It has 3 parts.
- **Recursive Rule:** defines the next term in relation to a previous term(s).
  ($a_n = a_{n-1} + d$ ***or*** $a_n = r \cdot a_{n-1}$)

The initial value
The 1st term.

Example: $a_1 = 4$
$a_n = a_{n-1} + 5$, where $n \geq 2$

The recursive rule
How to find the next term.

The first term for which to use the recursive rule.

Use the recursive formula given in the example above to find the first four terms.

4, 9, 14, 19