# 6-4 Rectangles

DEFINITION OF A RECTANGLE: A rectangle is a ___parallelogram___ with __4__ __right__ angles.

Properties of a Rectangle:	Example from the picture
1. Opposite sides are __parallel__ .	$\overline{AB} \parallel \overline{DC}$; $\overline{AD} \parallel \overline{BC}$
2. Opposite sides are __congruent__ .	$\overline{AB} \cong \overline{DC}$; $\overline{AD} \cong \overline{BC}$
3. Opposite angles are __congruent__ .	$\angle BAD \cong \angle BCD$ $\angle ADC \cong \angle ABC$
4. Consecutive angles are __supplementary__	ex. $m\angle ABC + m\angle BCD = 180$ there are 4 of these
5. The diagonals __bisect__ each other.	$AX = XC$; $DX = XB$
6. All four angles are __right__ angles.	$\angle BAD, \angle ADC, \angle DCB,$ & $\angle CBA$ are rt. ∠s
7. The diagonals are __congruent__.	$\overline{AC} \cong \overline{DB}$
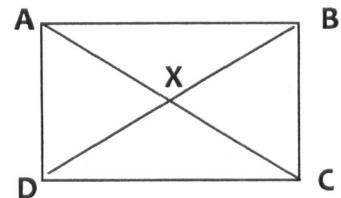
THESE PROPERTIES ARE WHAT SEPARATES THE RECTANGLE FROM A BASIC PARALLELOGRAM.

A ▢ B (with X at center) D C

1-6: Given rectangle RSTU, write an equation and solve for x. Then find each value. Circle your final answer.

1. If UZ = x + 21 & ZS = 3x – 15, find US.

R S (Z center, 3x – 15, x + 21) U T

$x+21=3x-15$
$36=2x$
$\boxed{x=18}$
$US = 4x+6$
$=4(18)+6=\boxed{78}$

2. If RZ = 3x + 8 & ZS = 6x – 28, find UZ.

R S (Z center, 3x + 8, 6x – 28) U T

$3x+8=6x-28$
$36=3x$
$\boxed{x=12}$
$UZ=3(12)+8 = \boxed{44}$

3. If RT = 5x + 8 & RZ = 4x + 1, find ZT.

R S (Z center, 4x + 1, 5x + 8) U T

$2(4x+1)=5x+8$
$8x+2=5x+8$
$3x=6$
$\boxed{x=2}$
$ZT=4(2)+1=\boxed{9}$

4. If m∠SUT = (3x + 6)° & m∠RUS = (5x – 4)°, find m∠SUT.

R S (Z center, (5x – 4)°, (3x + 6)°) U T

$5x-4+3x+6=90$
$8x+2=90$
$8x=88$
$\boxed{x=11}$
$m\angle SUT=3(11)+6=\boxed{39}$

5. If m∠SRT = (x + 9)° & m∠UTR = (2x – 44)°, find m∠UTR.

R S (Z center, (x + 9)°, (2x – 44)°) U T

$x+9=2x-44$
$\boxed{53=x}$
$m\angle UTR=2(53)-44=\boxed{62}$

6. If m∠RSU = (x + 41)° & m∠TUS = (3x + 9)°, find m∠RSU.

R S (Z center, (x + 41)°, (3x + 9)°) U T

$x+41=3x+9$
$32=2x$
$\boxed{x=16}$
$m\angle RSU=16+41=\boxed{57}$

7-12: Given rectangle GHJK, find each measure if m∠1 = 37°.

7. m∠2 = __53__   8. m∠3 = __37__   9. m∠4 = __37__

10. m∠5 = __53__   11. m∠6 = __106__   12. m∠7 = __74__
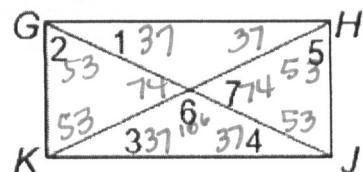
Coordinate Proof: *How would you prove a parallelogram is a rectangle? Think about the properties that would make the parallelogram a rectangle!*

The diagonals would be ≅ (dist. formula)

The angles would be right ∠s ∴ Consec. sides are ⊥ (slope formula)

13-15: Graph each parallelogram with the given vertices. Do a coordinate proof to determine whether the figure is a rectangle. (A picture alone is not sufficient evidence to justify your answer!)

13. B(-4, 3), G(-2, 4), H(1, -2), L(-1, -3)

$m\overline{BG} = \frac{1}{2}$

$m\overline{GH} = \frac{-6}{3} = -2$

$m\overline{HL} = \frac{1}{2}$

$m\overline{BL} = \frac{-6}{3} = -2$

$\overline{BG} \parallel \overline{HL}$ & $\overline{GH} \parallel \overline{BL}$
∴ it is a ▱
but if $\overline{BG} \perp \overline{GH} \perp \overline{HL} \perp \overline{BL}$, all 4 angles are right angles, so it is a rectangle!
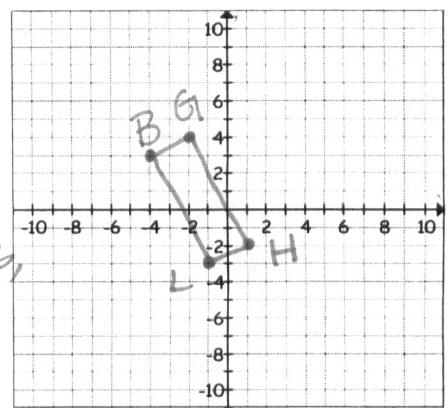
14. N(-4, 5), O(6, 0), P(3, -6), Q(-7, -1)

$NP = \sqrt{(3-(-4))^2 + (-6-5)^2} = \sqrt{(7)^2 + (-11)^2} = \sqrt{49+121} = \sqrt{170}$

$\overline{QO} = \sqrt{(-7-6)^2 + (-1-0)^2} = \sqrt{(-13)^2 + (-1)^2} = \sqrt{169+1} = \sqrt{170}$

Since NP = QO, the ▱ is a rectangle because the diagonals are ≅!
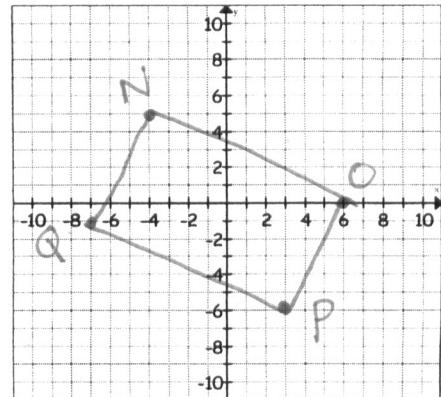
15. C(0, 5), D(4,7), E(5, 4), F(1, 2)

$m\overline{CD} = \frac{2}{4} = \frac{1}{2}$

$m\overline{DE} = \frac{-3}{1} = -3$

$m\overline{EF} = \frac{2}{4} = \frac{1}{2}$

$m\overline{CF} = \frac{-3}{1} = -3$

they are not ⊥, so CDEF is not a rectangle

✲ CDEF is just a ▱ because only 2 pair opposite sides are parallel!