9-4 Graphs of Rational Functions

Name Master
Date ______ Block ______

A ***RATIONAL FUNCTION*** is a function of the form $f(x) = \frac{p(x)}{q(x)}$, where $p(x)$ and $q(x)$ are polynomial functions with no common factors other than 1, and $q(x) \neq 0$.

Zeros

The ***zeros*** are the values of x where the ***numerator equals zero.***

Vertical Asymptotes

Set the denominator equal to zero and solve for x. There will be a vertical asymptote at each zero of $q(x)$.

Horizontal Asymptotes

- If the degree of $p(x)$ is ***less than*** the degree of $q(x)$, then there will be a horizontal asymptote at y = 0.
- If the degree of $p(x)$ is ***equal to*** the degree of $q(x)$, then there will be a horizontal asymptote at $y = \frac{\text{leading coefficient of } p(x)}{\text{leading coefficient of } q(x)}$.
- If the degree of $p(x)$ is ***greater than*** the degree of $q(x)$, then there will not be a horizontal asymptote.

Oblique/Slant Asymptotes

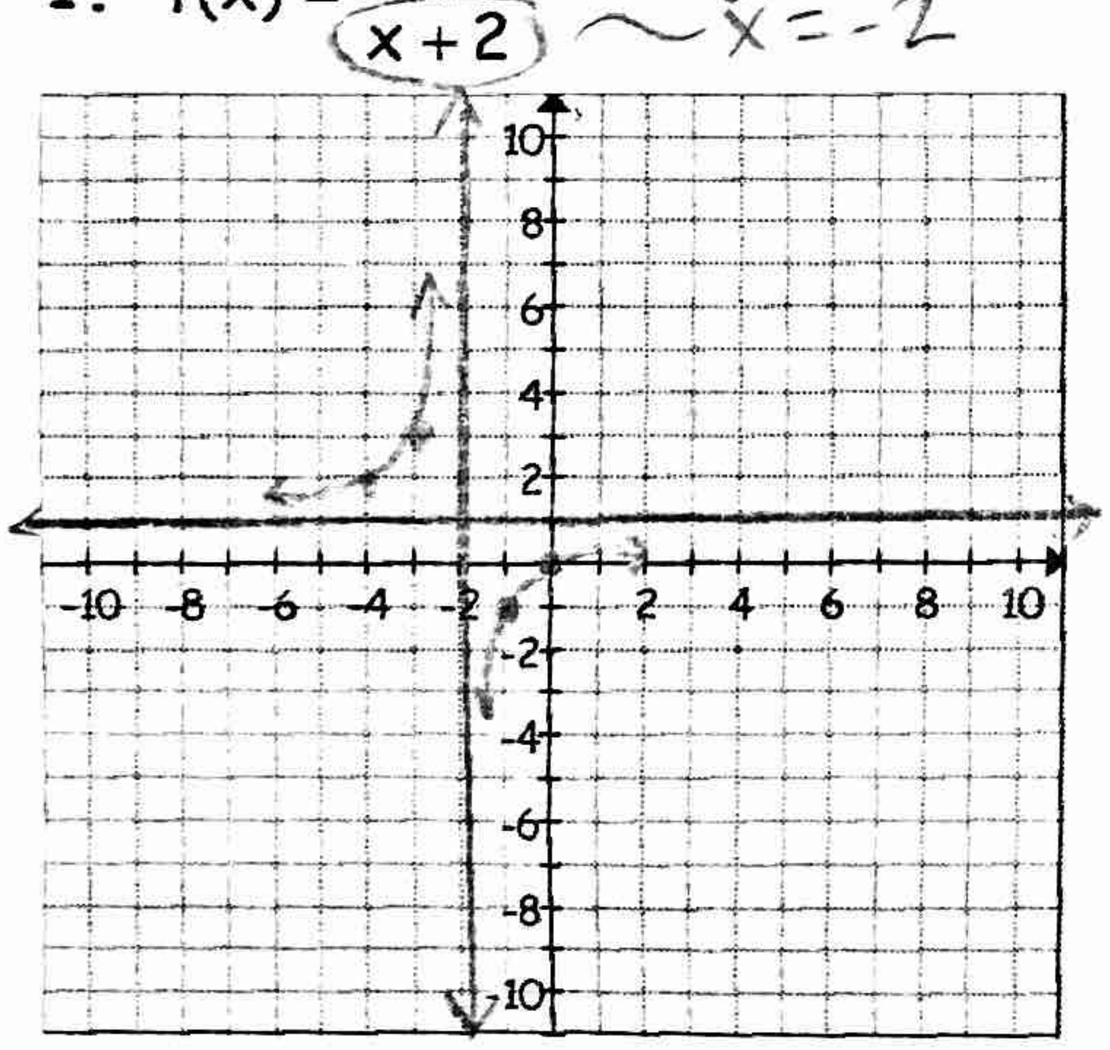
A ***slant asymptote*** exists when the degree of the numerator is ***exactly one greater*** than the degree of the denominator. Use long division to find the equation of the slant asymptote (ignore/discard the remainder).

Holes

A ***hole*** is a point of discontinuity and occurs when there is a ***common factor*** in the numerator and denominator Tell them they need to ✓

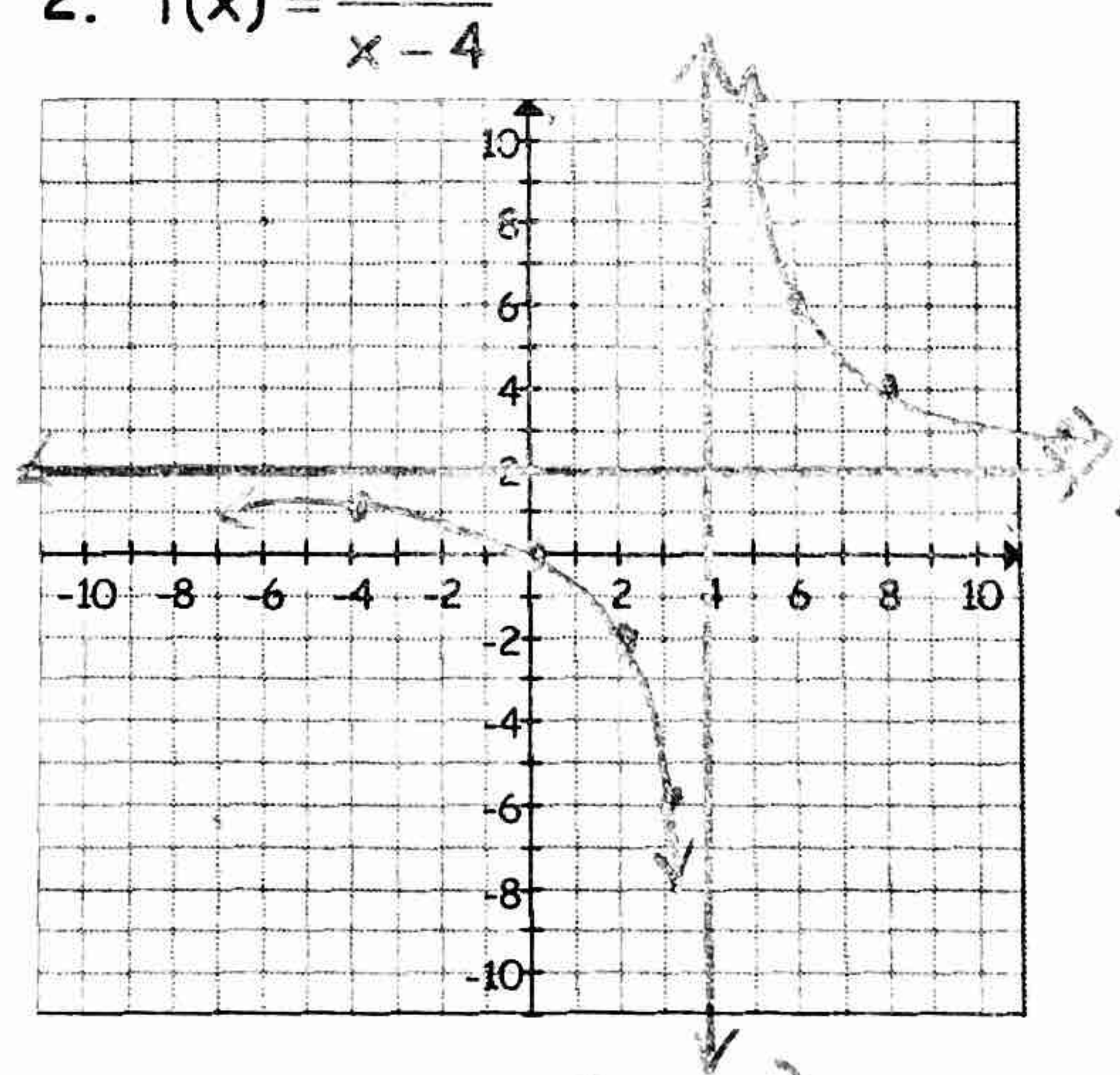
Graph each function and state the indicated parts.

1. $f(x) = \frac{x}{x+2}$ $y = \frac{1}{1} = 1$ $x = -2$



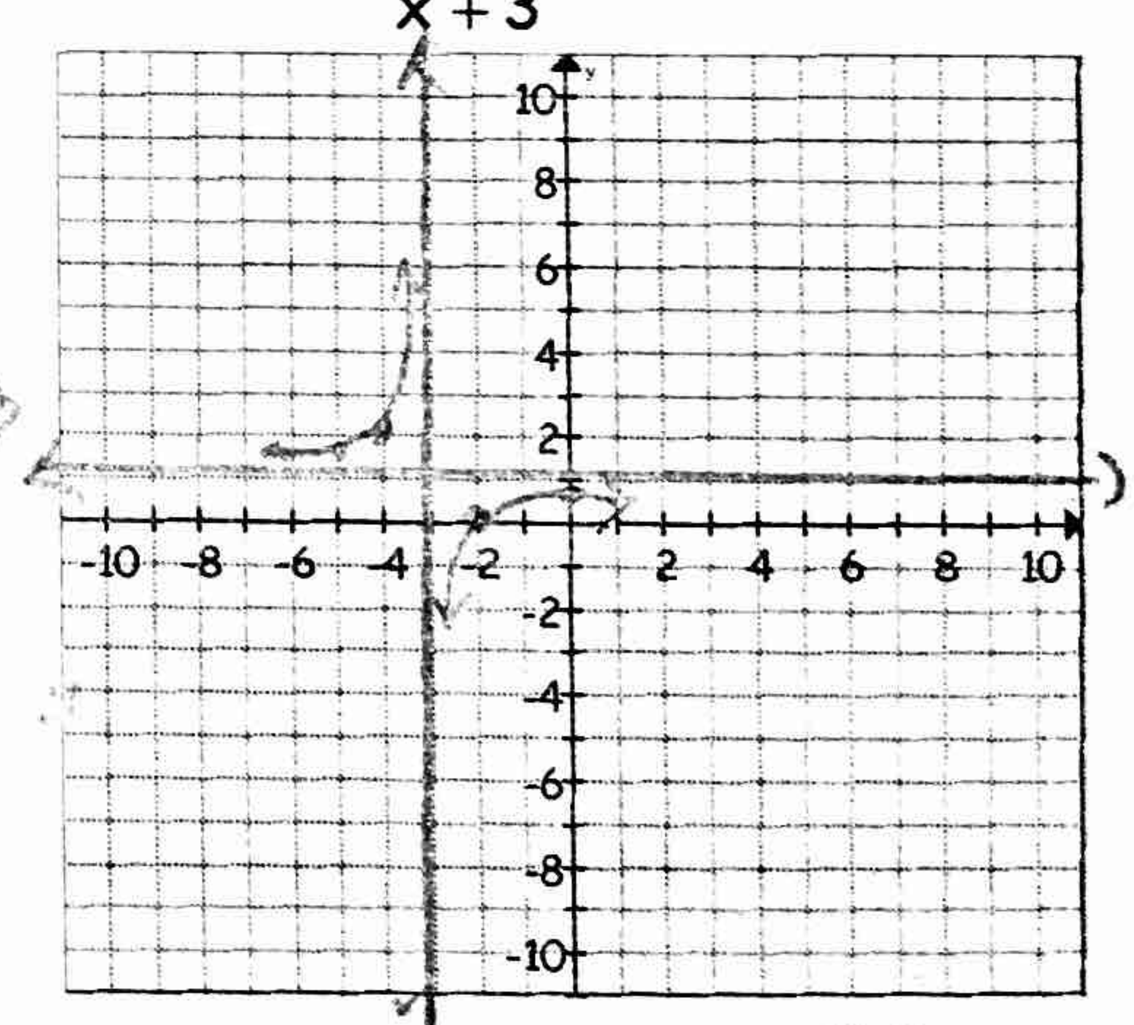
y-intercept: (0,0)
zeros: (0,0)
asymptotes: x = -2
y = 1
holes: none
domain: $(-\infty,-2)\cup(-2,\infty)$
range: $(-\infty,1)\cup(1,\infty)$

2. $f(x) = \frac{2x}{x-4}$



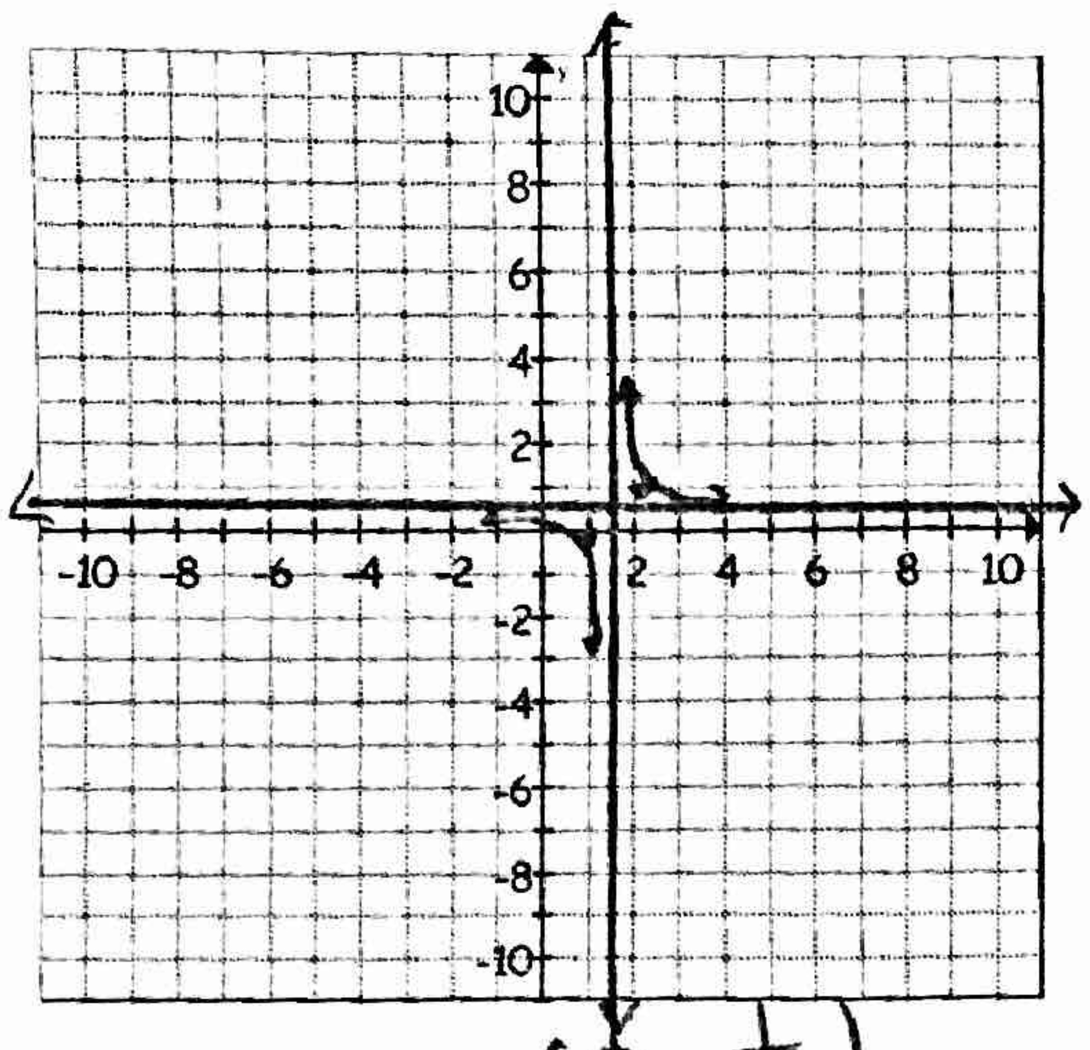
y-intercept: (0,0)
zeros: (0,0)
asymptotes: x = 4
y = 2
holes: none
domain: $(-\infty,4)\cup(4,\infty)$
range: $(-\infty,2)\cup(2,\infty)$

3. $f(x) = \frac{x+2}{x+3}$



y-intercept: $(0, \frac{2}{3})$
zeros: (-2, 0)
asymptotes: x = -3
y = 1
holes: none
domain: $(-\infty,-3)\cup(-3,\infty)$
range: $(-\infty,1)\cup(1,\infty)$

4. $f(x) = \frac{x-1}{2x-3}$



$\frac{-1}{-3}$

y-intercept: $(0, \frac{1}{3})$

zeros: $(1, 0)$

asymptotes: $x = 1.5$

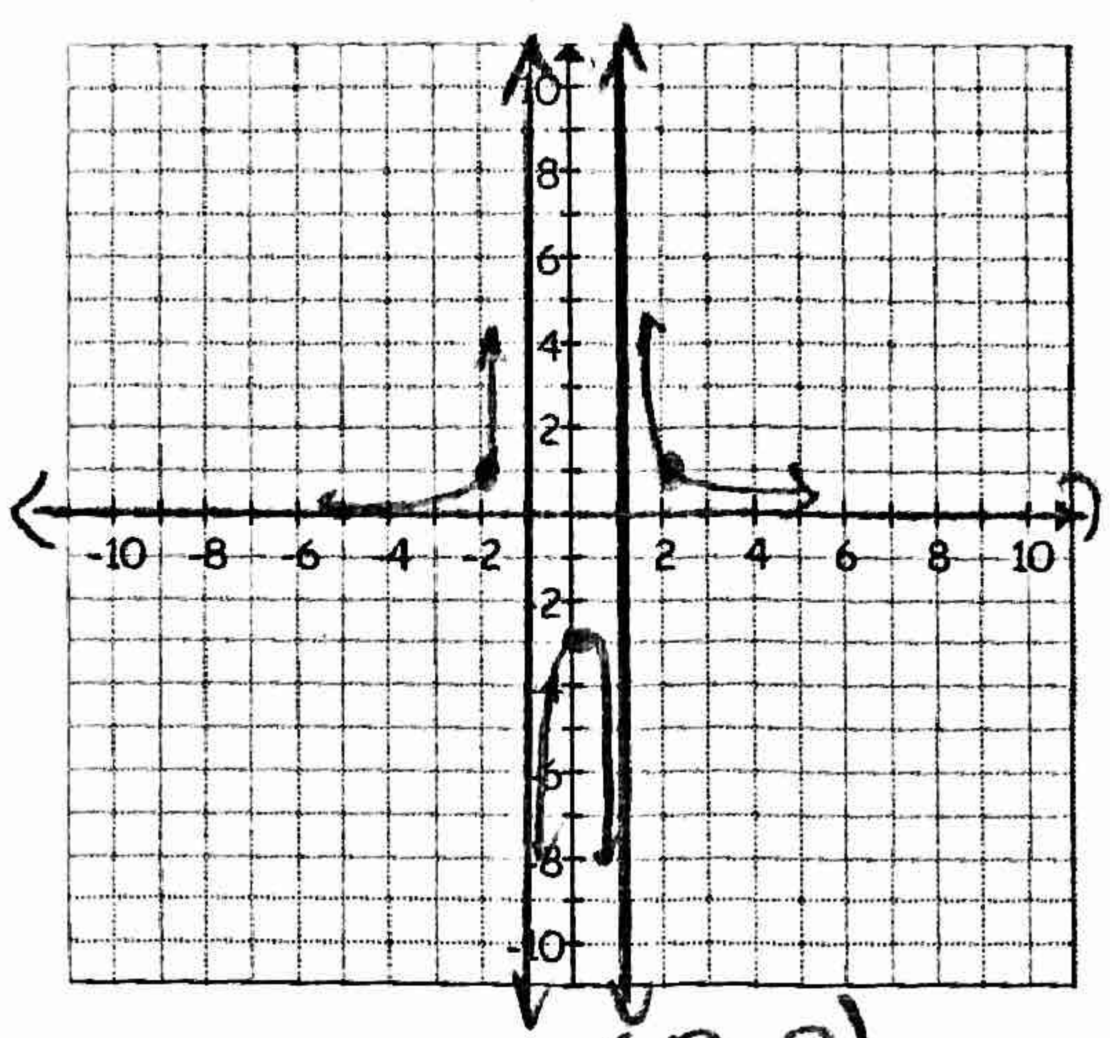
$y = \frac{1}{2}$

holes: none

domain: $(-\infty, 1.5) \cup (1.5, \infty)$

range: $(-\infty, \frac{1}{2}) \cup (\frac{1}{2}, \infty)$

5. $f(x) = \frac{3}{(x+1)(x-1)}$ $\quad \frac{3}{(1)(-1)} = -3$



y-intercept: $(0, -3)$

zeros: none

asymptotes: $x = -1, x = 1$

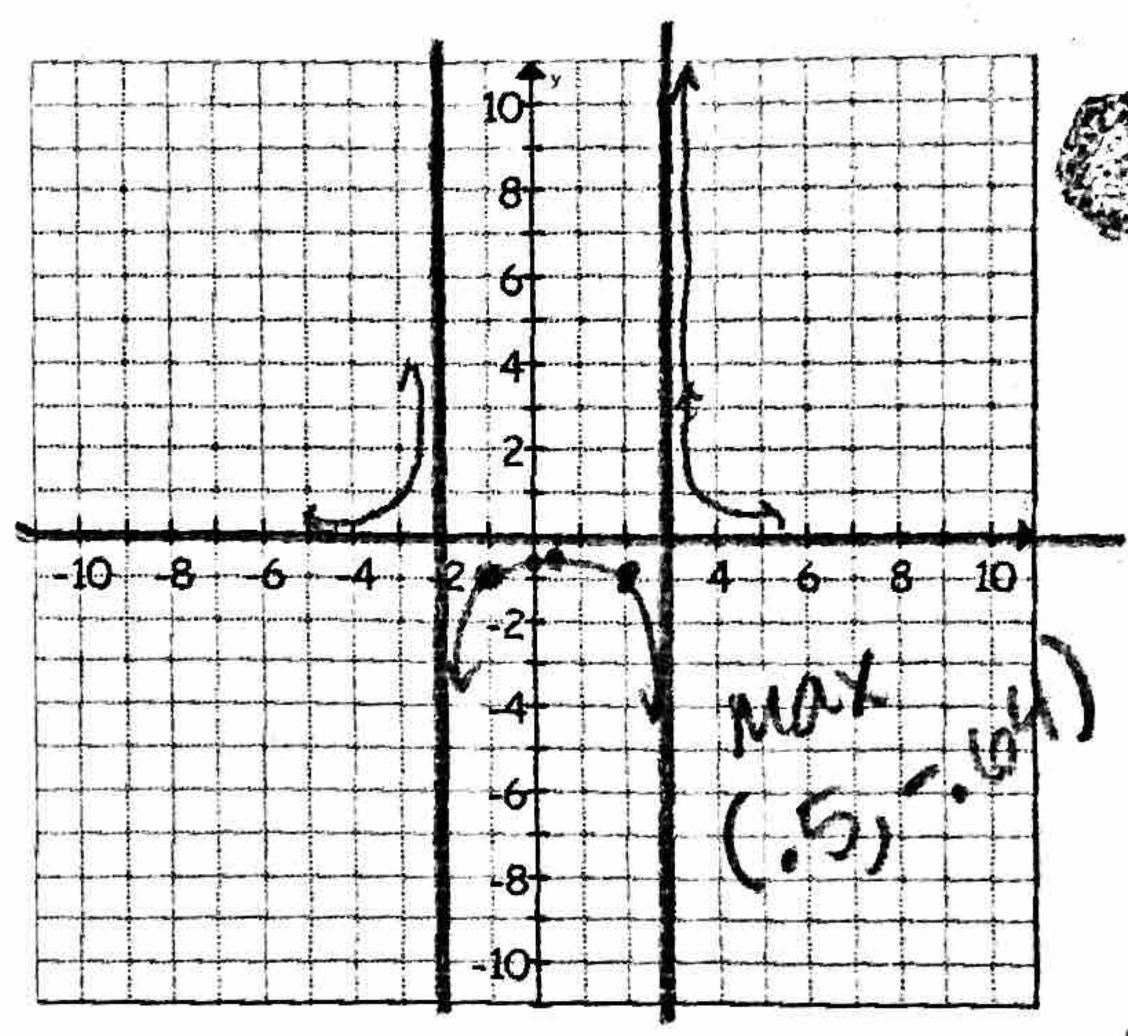
$y = 0$

holes: none

domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

range: $(-\infty, -3] \cup (0, \infty)$

6. $f(x) = \frac{4}{x^2 - x - 6}$ $\quad \frac{4}{(x-3)(x+2)}$



y-intercept: $(0, -\frac{2}{3})$ $\quad \frac{4}{-6} = -\frac{2}{3}$

zeros: none

asymptotes: $x = 3, x = -2$

$y = 0$

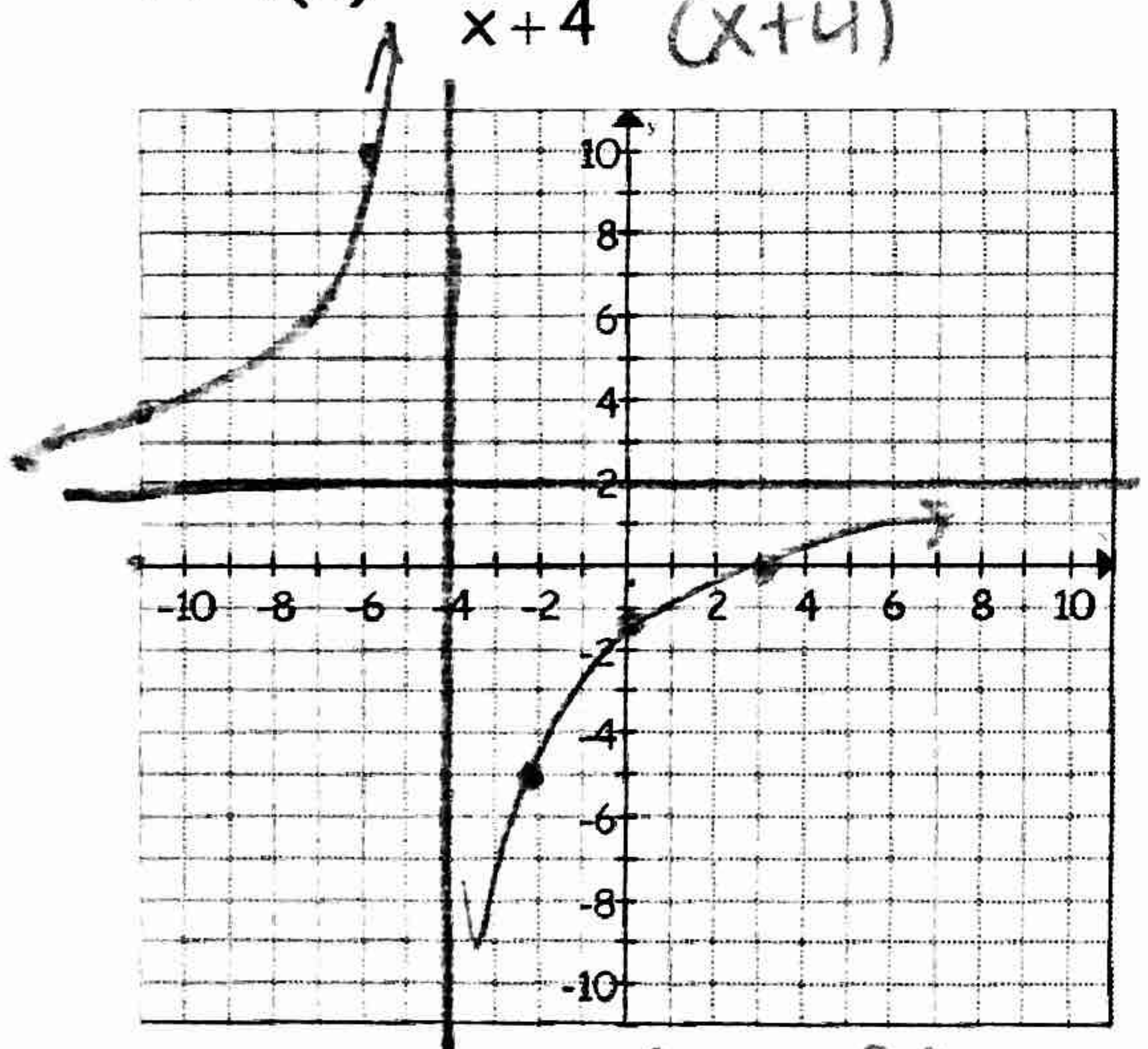
holes: none

domain: $(-\infty, -2) \cup (-2, 3) \cup (3, \infty)$

*range: $(-\infty, -.64] \cup (0, \infty)$

*calc. the max.

7. $f(x) = \frac{2x-6}{x+4}$ $\quad \frac{2(x-3)}{(x+4)}$



$\frac{-6}{4}$

y-intercept: $(0, -\frac{3}{2})$

zeros: $(3, 0)$

asymptotes: $x = -4$

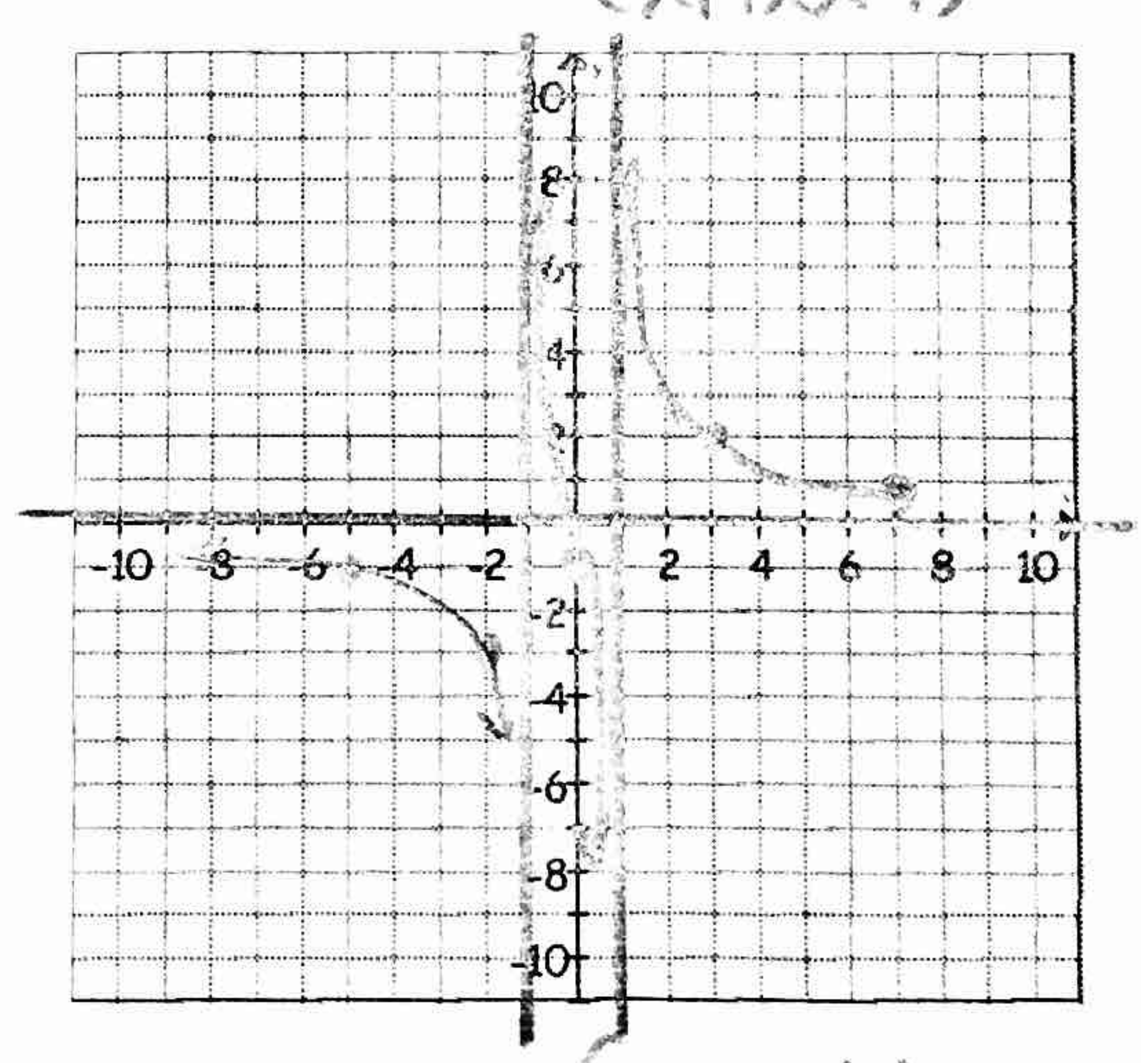
$y = 2$

holes: none

domain: $(-\infty, -4) \cup (-4, \infty)$

range: $(-\infty, 2) \cup (2, \infty)$

8. $f(x) = \frac{5x+1}{x^2 - 1}$ $\quad \frac{(5x+1)}{(x+1)(x-1)}$



y-intercept: $(0, -1)$

zeros: $(-\frac{1}{5}, 0)$

asymptotes: $x = -1, x = 1$

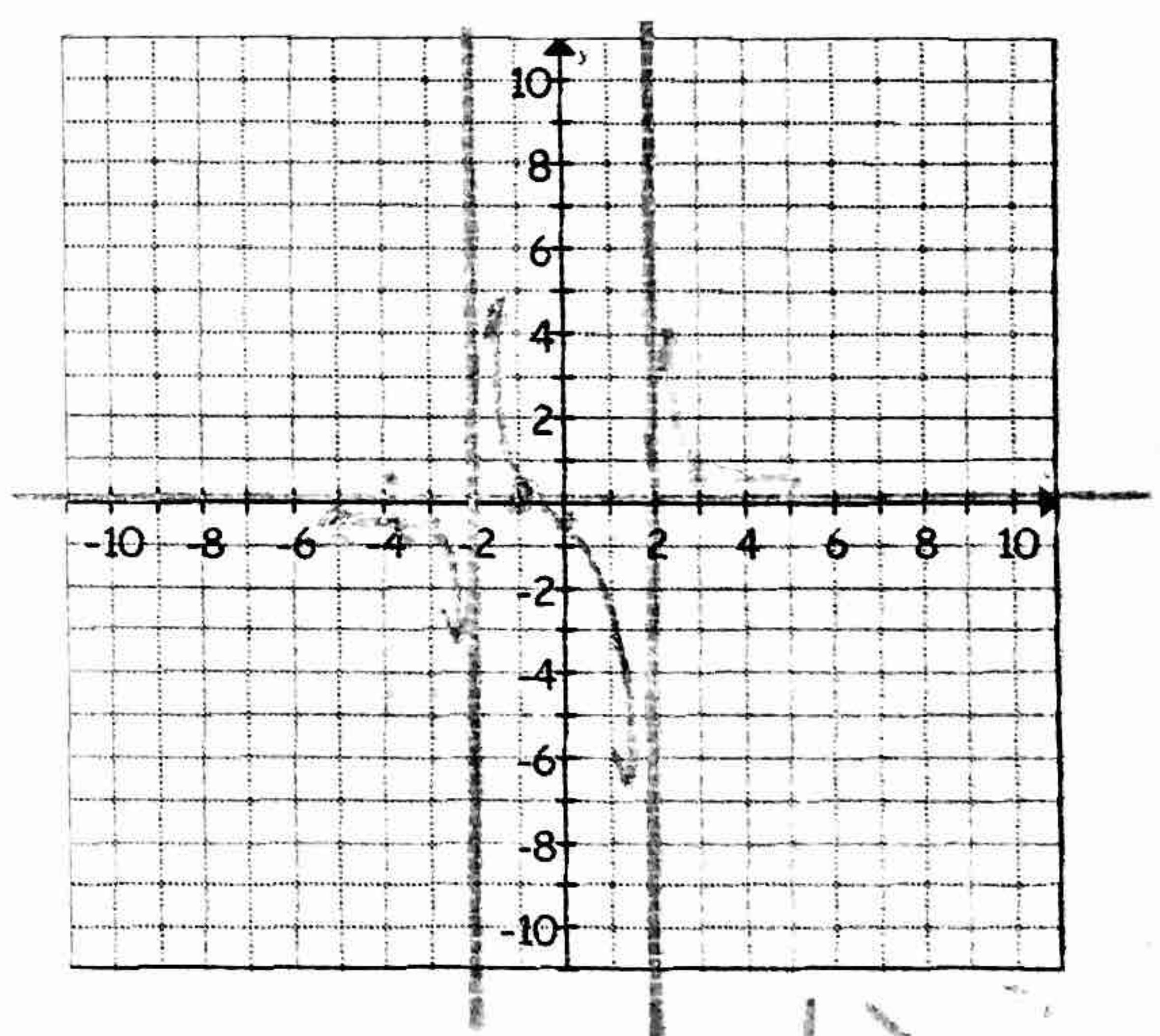
$y = 0$

holes: none

domain: $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

range: $(-\infty, \infty)$

9. $f(x) = \frac{x+1}{x^2 - 4}$



y-intercept: $(0, -\frac{1}{4})$

zeros: $(-1, 0)$

asymptotes: $x = 2, x = -2$

$y = 0$

holes: none

domain: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

range: $(-\infty, \infty)$