Day 03 Polynomial Functions and End Behavior Practice

Name Master Ø "

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Polynomial in One Variable	A polynomial of degree n in one variable x is an expression of the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, where the coefficients $a_{n-1}, a_{n-2}, a_{n-3} \ldots a_0$ represent real numbers, a_n is not zero, and n represents a positive integer.
Degree of a Polynomial	the term with the greatest exponent determines the degree
Leading Coefficient	the coefficient of the term with the highest degree.
Polynomial Function	A polynomial function of degree n can be described by an equation of the form $P(x) = a_n x^n + a_{n-1}x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$, where the coefficients $a_{n-1}, a_{n-2}, a_{n-3}, \ldots, a_0$ represent real numbers, a_n is not zero, and n represents a positive integer.
Exponents, Coefficients, & Constants	Will always be whole numbers and never imaginary numbers
General/Standard form:	The terms are written in descending order
Evaluating a function:	If you know an element in the domain of any polynomial function, you can find the corresponding value in the range.
Graphs of Polynomial Functions	The general shapes of the graphs of polynomial functions shows the maximum number of times the graph of each function may intersect the x-axis. This is the same number as the degree of the polynomial.

Example 1: What are the degree and leading coefficient of $3x^2 - 2x^4 - 7 + x^3$ a. Rewrite the expression in standard form (the powers of x are in decreasing order): $-2x^4 + x^3 + 3x^2 - 7$ b. This is a polynomial in one variable. The degree is 4, and the leading coefficient is −2.		
Example 2: If $f(x) = x^3 + 2x^2 - 10x + 20$, find $f(-5)$	$f(x) = x^3 + 2x^2 - 10x + 20$ $f(-5) = (-5)^3 + 2(-5)^2 - 10(-5) + 20$ $= -125 + 50 + 50 + 20$ $= -5$	Original function Replace x with −5. Evaluate. Simplify.
Example 3: Find $g(a^2 - 1)$ if $g(x) = x^2 + 3x - 4$	$g(x) = x^2 + 3x - 4$ $g(a^2 - 1) = (a^2 - 1)^2 + 3(a^2 - 1) - 4$ $= a^4 - 2a^2 + 1 + 3a^2 - 3 - 4$ $= a^4 + a^2 - 6$	Original function Replace x with $a^2 - 1$. Evaluate. Simplify.

YOUR TURN TO PRACTICE ☺

1-6: Given each polynomial, do the following:
a. Write each polynomial in standard form, b. State the degree, and c. State the leading coefficient of each

1. $3x^4 + 6x^3 - x^2 + 12$
a. ↓
b. 4
c. 3

2. $100 - 5x^3 + 10x^7$
a. $10x^7 - 5x^3 + 100$
b. 7
c. 10

3. $4x^6 + 6x^4 + 8x^8 - 10x^2 + 20$
a. $8x^8 + 4x^6 + 6x^4 - 10x^2 + 2$
b. 8
c. 8

4. $4x^2 - 3x^3 + 16x - 2$
a. $-3x^3 + 4x^2 + 16x - 2$
b. 3
c. −3

5. $8x^3 - 9x^5 + 4x^2 - 36$
a. $-9x^5 + 8x^3 + 4x^2 - 36$
b. 5
c. −9

6. $\frac{x^2}{18} - \frac{x^6}{25} + \frac{x^3}{36} - \frac{1}{72}$
a. $-\frac{1}{25}x^6 + \frac{1}{36}x^3 + \frac{1}{18}x^2 - \frac{1}{72}$
b. 6
c. $-\frac{1}{25}$

7-9: Given $f(x) = 3x^2 - 9$ and $g(x) = 4x^3 - 3x^2 + 2x - 1$, find each value.

7. f(3a)
$3(3a)^2 - 9$
$3(9a^2) - 9$
$27a^2 - 9$

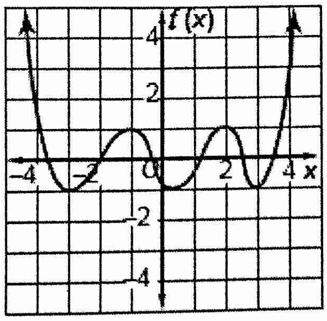
8. g(-4)
$4(-4)^3 - 3(-4)^2 + 2(-4) - 1$
$4(-64) - 3(16) - 8 - 1$
$-256 - 48 - 9$
-313

9. f(x + 2)
$3(x+2)^2 - 9$
$3(x^2 + 4x + 4) - 9$
$3x^2 + 12x + 12 - 9$
$3x^2 + 12x + 3$

10-18: Given each graph, do the following:

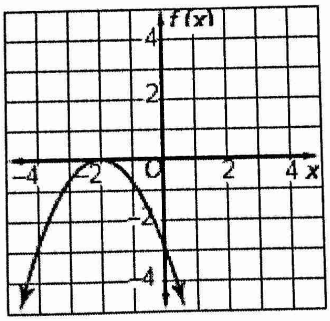
a. Describe the end behavior
b. State the domain and range in interval notation
c. Determine whether it represents an odd-degree or an even-degree function, and
d. state the number of real zeroes.

10.



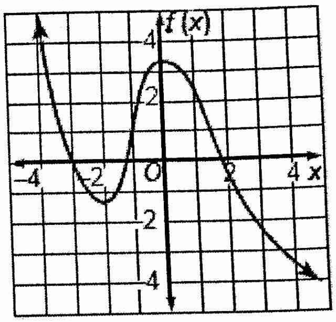
a. As $x\to-\infty$, $f(x)\to+\infty$
As $x\to+\infty$, $f(x)\to+\infty$
b. D: $(-\infty,\infty)$
R: $[-1,\infty)$
c. even degree
d. 6

11.



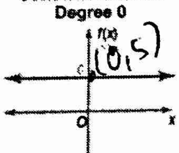
a. As $x\to-\infty$, $f(x)\to-\infty$
As $x\to+\infty$, $f(x)\to-\infty$)
b. D: $(-\infty,\infty)$
R: $(-\infty,0]$
c. even degree
d. 2 (double root)

12.



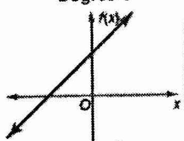
a. As $x\to-\infty$, $f(x)\to+\infty$
As $x\to+\infty$, $f(x)\to-\infty$
b. D: $(-\infty,\infty)$
R: $(-\infty,\infty)$
c. odd degree
d. 3

13. Constant function Degree 0



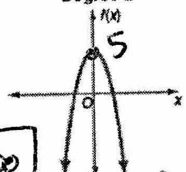
a. As $x\to-\infty$, $f(x)=5$
As $x\to+\infty$, $f(x)=5$
b. D: $(-\infty,\infty)$
R: $\{5\}$
c. N/A
d. 0

14. Linear function Degree 1



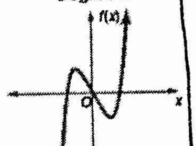
a. As $x\to-\infty$ $f(x)\to-\infty$
As $x\to+\infty$ $f(x)\to+\infty$
b. D: $(-\infty,\infty)$
R: $(-\infty,\infty)$
c. odd
d. 1

15. Quadratic function Degree 2



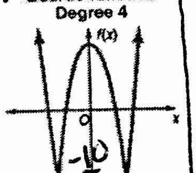
a. As $x\to-\infty$ $f(x)\to-\infty$
As $x\to+\infty$ $f(x)\to-\infty$
b. D: $(-\infty,\infty)$
R: $(-\infty,5]$
c. even
d. 2

16. Cubic function Degree 3



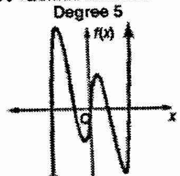
a. As $x\to-\infty$ $f(x)\to-\infty$
As $x\to+\infty$ $f(x)\to+\infty$
b. D: $(-\infty,\infty)$
R: $(-\infty,\infty)$
c. odd
d. 3

17. Quartic function Degree 4



a. As $x\to-\infty$ $f(x)\to+\infty$
As $x\to+\infty$ $f(x)\to+\infty$
b. D: $(-\infty,\infty)$
R: $[-10,\infty)$
c. even
d. 4

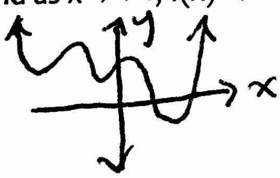
18. Quintic function Degree 5



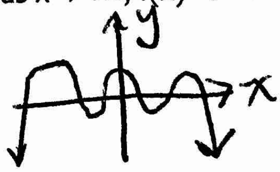
a. As $x\to-\infty$, $f(x)\to-\infty$
As $x\to+\infty$, $f(x)\to+\infty$
b. D: $(-\infty,\infty)$
R: $(-\infty,\infty)$
c. odd
d. 5

19-22: Create a sketch of a function that has the given end behavior.

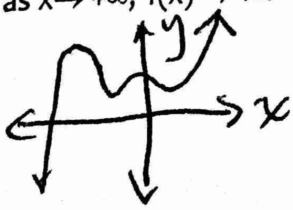
19. As $x\to-\infty$, $f(x)\to+\infty$
and as $x\to+\infty$, $f(x)\to+\infty$



20. As $f(x)\to-\infty$, $x\to-\infty$
and as $x\to+\infty$, $f(x)\to-\infty$



21. As $x\to-\infty$, $f(x)\to-\infty$
and as $x\to+\infty$, $f(x)\to+\infty$



22. As $f(x)\to+\infty$, $x\to-\infty$
and as $x\to+\infty$, $f(x)\to-\infty$