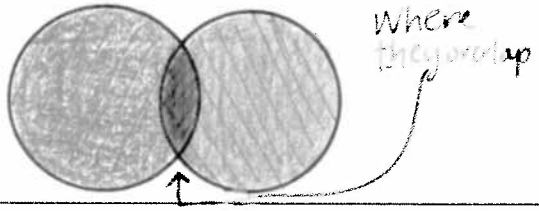
1-6 Solving Compound & Absolute Value Inequalities

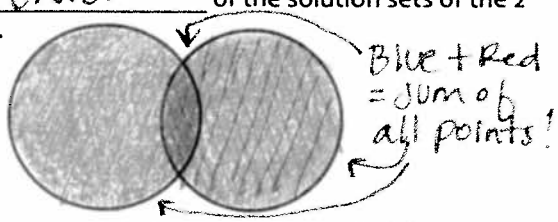
Compound Inequality: ♥Consists of 2 inequalities joined by the word AND or the word OR.

♥To solve a compound inequality, you must solve each PART of the inequality.

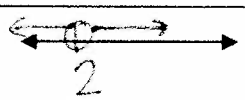
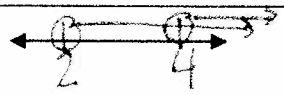
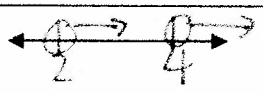
♥The graph of a compound inequality containing the word **AND** is the INTERSECTION of the solution sets of the 2 inequalities.



♥The graph of a compound inequality containing the word **OR** is the UNION of the solution sets of the 2 inequalities.



What are the differences between the following? Graph and then give the solution in set-builder notation.

1. $x = 2$	2. $x < 2$	3. $x > 2$	4. $x > 2$ AND $x < 4$
			
$\{2\}$	$x < 2$	$x > 2$	$2 < x < 4$
5. $x > 2$ OR $x < 4$	**6.** $x < 2$ AND $x > 2$	**7.** $x > 2$ AND $x > 4$	**8.** $x > 2$ OR $x > 4$
			
∞ SOL. / $\mathbb{R}$	$\emptyset$	$x > 4$	$x > 2$

Solving Conjunctions (use AND): Both conditions must be true. *Where do they overlap?*

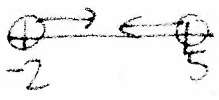
9. $4 < 3 + x < 10$

$-3 \quad -3 \quad -3$

$\boxed{1 < x < 7}$

10. $2x < 10$ and $x > -2$

$x < 5 \quad x > -2$



$\boxed{-2 < x < 5}$

11. $-4 < 2 - x < 6$

$-2 \quad -2 \quad -2$

$-6 < -x < 4$

$6 > x > -4$

$\boxed{-4 < x < 6}$

12. $35 - 5x \leq 0$ and $5x + 6 \geq -14$

$-5x \leq -35 \quad 5x \geq -20$

$x \geq 7 \quad x \geq -4$

FA: $\boxed{x \geq 7}$

Solving Disjunctions (use OR): All solutions are part of the solution. *What is the set of all points?*

13. $4 < 3 + x$ or $x - 5 < 1$

$1 < x \quad x < 6$

$x > 1$

FA: $\boxed{\mathbb{R}}$

14. $x + 1 < -5$ or $x + 1 > 3$

$x < -6 \quad x > 2$

$\boxed{x < -6 \text{ or } x > 2}$

15. $-7x \leq -28$ or $7x \geq 7$

$\frac{-7}{-7} \quad \frac{-7}{-7} \quad \frac{7}{7} \quad \frac{7}{7}$

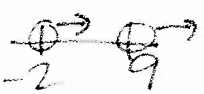
$x \geq 4 \quad x \geq 1$

FA: $\boxed{x \geq 1}$

16. $2x - 3 > 15$ or $3 - 7x < 17$

$2x > 18 \quad -7x < 14$

$x > 9 \quad x > -2$

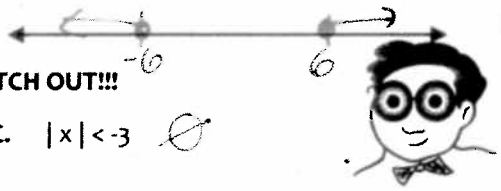


FA: $\boxed{x > -2}$

Remember: The absolute value of x: $|x|$ = the distance the number is from zero on a number line.
Since this value is a distance it can never be negative!

Intro: If $|x| = 6$ then $x =$ 6 or −6

If $|x| \geq 6$ then $x \geq 6$ or $x \leq -6$



What is the difference between the following absolute value problems? WATCH OUT!!!

A. $|x| = -3$ Ø B. $|x| > -3$ ℝ C. $|x| < -3$ Ø

Steps to Solving Absolute Value Inequalities:

1. Isolate the absolute value on the left side of the inequality.
2. Set up two inequalities:
   - Drop the absolute value and keep everything the same.
   - Drop the absolute value and switch the sign of the inequality AND the expression on the right.
3. Solve both inequalities and check.
4. Graph your result to help write your final answer.
5. Write your answer according to the type of solution (AND or OR).
   - $|x| < a$ "LessthAND" $-a < x < a$
   - $|x| > b$ "GreatOR" $x < -b$ OR $x > b$

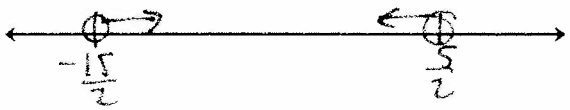
Example: $|4x + 10| < 20$ AND!

$4x + 10 < 20$ and $4x + 10 > -20$

$4x < 10$ and $4x > -30$

$x < \frac{10}{4}$ and $x > -\frac{30}{4}$

$x < \frac{5}{2}$ and $x > -\frac{15}{2}$



(shade between, therefore, it is an "and" answer!)

$$-\frac{15}{2} < x < \frac{5}{2}$$

Solve each absolute value inequality.

1. $|x| > -10$ ℝ

If you did the work: $x > -10$ OR $x < 10$

OR: Jump all pts = ℝ! ∴ ℝ

2. $|x| < -10$ Ø

If you did the work: $x < -10$ AND $x > 10$

They don't overlap! ∴ Ø

3. $|2x - 5| > 9$ OR

$2x - 5 > 9$ or $2x - 5 < -9$

$2x > 14$ $2x < -4$

$x > 7$ $x < -2$

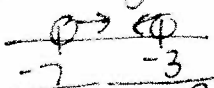
$x < -2$ or $x > 7$

4. $3|y + 5| < 6$

$|y + 5| < 2$ AND

$y + 5 < 2$ $y + 5 > -2$

$y < -3$ $y > -7$



$-7 < y < -3$

***5.** $|x + 2| \leq 2x + 7$ AND

$x + 2 \leq 2x + 7$ $x + 2 \geq -2x - 7$

$-x \leq 5$ $3x \geq -9$

$x \geq -5$ $x \geq -3$

FA: $x \geq -3$

***6.** $|x + 2| - 5 \leq 2x + 7$

$|x + 2| \leq 2x + 12$ AND

$x + 2 \leq 2x + 12$ $x + 2 \geq -2x - 12$

$-x \leq 10$ $3x \geq -14$

$x \geq -10$ $x \geq -\frac{14}{3}$

FA: $x \geq -\frac{14}{3}$

***7.** $-|x - 7| + 5 \geq 3x - 2$

$-|x - 7| \geq 3x - 7$

$|x - 7| \leq -3x + 7$ AND

$x - 7 \leq -3x + 7$ $x - 7 \geq 3x - 7$

$4x \leq 14$ $-7 \geq 2x - 7$

$x \leq \frac{14}{4}, \frac{7}{2}$ $0 \geq 2x$

$x \leq 0$

$x \leq 0$