## Day 04 Properties of Logarithms

| PRODUCT PROPERTY | QUOTIENT PROPERTY | POWER PROPERTY |
| :---: | :---: | :---: |
| Introduction: $\begin{gathered} \log _{3}(9 \cdot 27)=\log _{3}\left(3^{2} \cdot 3^{3}\right)= \\ \log _{3} 3^{2+3}=2+3=5 \end{gathered}$ <br> When you multiply monomials with like bases, you ADD the exponents!! | Introduction: $\begin{gathered} \log _{3}\left(\frac{81}{27}\right)=\log _{3}\left(\frac{3^{4}}{3^{3}}\right)= \\ \log _{3} 3^{4-3}=4-3=1 \end{gathered}$ <br> When you divide monomials with like bases, you SUBTRACT the exponents! | Introduction: $\begin{aligned} & \log _{3} 9^{4}=\log _{3}\left(3^{2}\right)^{4}= \\ & \log _{3} 3^{204}=2 \cdot 4=8 \end{aligned}$ <br> When you raise a monomial to a power, you MULTIPLY the exponents! |
| Conclusion: <br> Since a logarithm is an exponent, then you "expand" and add the logs. | Conclusion: <br> Since a logarithm is an exponent, then you "expand" and subtract the logs. | Conclusion: <br> Since a logarithm is an exponent, then you multiply the power times the log. |
| PRODUCT Property $\begin{gathered} \log _{b} m n=\log _{b} m+\log _{b} n \\ m>0, n>0, \& b \neq 1 \end{gathered}$ | QUOTIENT Property $\begin{aligned} \log _{b} \frac{m}{n} & =\log _{b} m-\log _{b} n \\ m & >0, n>0, \& b \neq 1 \end{aligned}$ | POWER Property $\log _{b} m^{p}=p \log _{b} m$ $p$ must be real, $m>0, b>0 \text { and } b \neq 1$ |
| Example: $\begin{gathered} \log 5+\log 3=\log x \\ \log (5 \cdot 3)=\log x \\ \log 15=\log x \\ 15=x \rightarrow x=15 \end{gathered}$ | Example: $\begin{gathered} \log _{5} 8-\log _{5} 2=\log _{5} 2 x \\ \log _{5}\left(\frac{8}{2}\right)=\log _{5} 2 x \\ \frac{8}{2}=2 x \rightarrow 4=2 x \rightarrow x=2 \end{gathered}$ | Example: <br> Evaluate $\log _{3} 9^{4}$ $\log _{3} 9^{4}=4 \log _{3} 9$, <br> since $\log _{3} 9=2$ (because $3^{2}=9$ ) $\log _{3} 9^{4}=4 \cdot 2=8$ |
| 1-9: Expand each expression. |  |  |
| 1. $\log _{6} 3 x$ | 2. $\log _{2} \frac{x}{5}$ | 3. $\log x y^{2}$ |
| 4. $\log _{4} \frac{x y}{3}$ | 5. $\log _{3} \sqrt{x} y z$ | 6. $\log _{5} 2 \sqrt{x}$ |
| 7. $\log \frac{x^{2}}{4}$ | 8. $\log \frac{10}{\sqrt{x}}$ | 9. $\log _{2} \frac{x^{2} y}{z}$ |

10-15: Condense each expression.

| 10. $\log _{3} 7-\log _{3} x$ | 11. $2 \log _{5} x+\log _{5} 3$ | 12. $\log _{4} 5+\log _{4} x+\log _{4} y$ |
| :--- | :--- | :--- |
| 13. $\frac{1}{2} \log x-\log 4$ | 14. $\frac{2}{3} \log _{2} x-3 \log _{2 y}$ | 15. $\log _{3} 4+2 \log _{3} x-\log _{3} 5$ |
| The pH of a patient's blood can be calculated using the Henderson-Hasselbach Formula, <br> pH $=6.1+\log \frac{B}{C}$, where $B$ is the <br> concentration of bicarbonate and $C$ is the concentration of carbonic |  |  |
| acid. The normal pH blood is approximately 7.4. |  |  |

19-24: Solve each equation. Round to three decimal places when necessary.
19. $\log _{2} x+\log _{2}(x+1)=1$
21. $\log (x+2)+\log (x-3)=\log (x+29)$
23. $3 \log _{5}\left(x^{2}+9\right)-6=0$
24. $\log _{2} x=5 \log _{2} 2-\log _{2} 8$

