SOL AII.2 The student will investigate and apply the properties of arithmetic and geometric sequences and series to solve real-world problems, including writing the first *n* terms, finding the n^{th} term, and evaluating summation formulas. Notation will include $\sum_{i=1}^{n} X_i$ and a_n.

Key

Hints and Notes

- Be familiar with the formulas that will be on the Algebra II SOL Formula Sheet.
- In an **ARITHMETIC** sequence a ***common difference*** is **ADDED** to each term to find the subsequent term.
- In a **GEOMETRIC** sequence each term is **MULTIPLIED** by a ***common ratio*** to find the subsequent term.
- If you get stuck using the formulas, use a calculator to find the needed term, sum, etc.

PRACTICE AII.2

1. What is the sum of the series defined by $\sum_{n=0}^{4}(3-2n)$

arithmetic

$a_1 = 3-2(0) = 3$
$a_n = 3-2(4) = -5$
$n = 4-0+1 = 5$

A -5
B -3
C -1
D 0

$S_5 = \frac{5}{2}(3-5)$
$S_5 = \frac{5}{2}(-2) = -5$

2. Two geometric means between 2 and 54 are

A 4 *and* 12
B 6 *and* 12
C 6 *and* 18
D 12 *and* 18

$2, \underline{6}, \underline{18}, 54$
$a_1 \quad a_4$
$a_4 = a_1 r^{4-1}$
$54 = 2r^3$
$27 = r^3 \Rightarrow r = 3$

3. If $a_n = 2^{n-1}$, which number represents $a_4 = ?$

A 15
B 8
C 7
D 6

$a_4 = 2^3$
$a_4 = 8$

4. What are two arithmetic means between 3 and 24?

A 8 *and* 12
B 8 *and* 16
C 9 *and* 16
D 10 *and* 17

$3, \underline{10}, \underline{17}, 24$
$a_1 \quad a_4$
$a_4 = a_1 + (4-1)d$
$24 = 3 + 3d$
$21 = 3d \Rightarrow d = 7$

5. What is the value of $\sum_{n=1}^{6} 2^n$?

geometric

A 62
B 126
C 128
D 252

$a_1 = 2^1 = 2$
$n = 6-1+1 = 6$
$r = 2$
$S_6 = \frac{2(1-2^6)}{1-2}$
$S_6 = 126$

6. If $a_n = 1+\frac{1}{n}$, then what is a_9 ?

A $\frac{11}{10}$
B $\frac{10}{9}$
C $\frac{9}{8}$
D $\frac{3}{2}$

$a_9 = 1 + \frac{1}{9}$
$a_9 = 1\frac{1}{9} = \frac{10}{9}$

TEI (Technology Enhanced Item): Free-Response - For free-response questions, type your answer in the box. Be sure your answer is in appropriate form - simplest fraction, decimal, etc.

7. Driving a piling into a harbor bottom, a pile driver sinks the piling 24 inches on the first stroke (a_1), 18 inches on the second stroke (a_2), and $13\frac{1}{2}$ inches on the third stroke (a_3). If the sequence is continued, how far will the piling be driven down on the 5th stroke?

$24, 18, 13.5, \underline{10.125}, \underline{7.59375}$
a_1

$r = \frac{18}{24} = 0.75$
$r = \frac{13.5}{18} = 0.75$

$\boxed{7.59375}$

$7\frac{19}{32}$

SOL AII.9 The student will collect and analyze data, determine the equation of the curve of best fit, make predictions, and solve real-world problems, using mathematical models. Mathematical models will include polynomial, exponential, and logarithmic functions.

Hints and Notes

Key

To determine the best model for a set of data using a graphics calculator:

- Put the x's in L_1 and y's in L_2 and graph the scatter plot (2nd STAT PLOT) –
- Adjust the window as needed (ZOOM>Stat)
- Use the shape of the data to determine which may be the best model,
- Then use STAT>CALC and one of the regression models (#4 Linear, #5 Quadratic, #6 Cubic, etc.), to find a function that best fits the data.

PRACTICE AII.9

1. In 1940, the life expectancy at birth in the general public was 62.9 years. By 1980 it had risen to 73.7 years. Assuming a linear relation, which is the best prediction of life expectancy in the year 2000?

(1940, 62.9)
(1980, 73.7)

A 76.4
B 79.1
C 79.9
D 84.5

$y = 0.27x - 460.9$
$y = 0.27(2000) - 460.9$
$y = 79.1$

2. Which is the best quadratic model for the data {(-4, 9), (-2, 1), (-1, 0), (0, 1), (2, 8)} ?

$y = 0.94x^2 + 1.73x + 0.83$

A $y = x^2$
B $y = x^2 + 1$
C $y = 2x^2 + x + 1$
D $y = x^2 + 2x + 1$

3. Which type of function would best model the data?

x	0	4	8	12	16	20
y	2	6	14	26	54	110

A Quadratic
B Cubic
C Exponential
D Logarithmic

4. What is the best model equation for the data

x	0	4	8	12	16	20
y	2	6	14	26	54	110

see #3

A $y = 2.61x + 5.87$
B $y = 2.46(1.21)^x$
C $y = 1.21(2.46)^x$
D $y = 2.61x^2 + 5.87$

5. What is your best prediction for y when $x = 30$?

see #3 and #4

x	0	4	8	12	16	20
y	2	6	14	26	54	110

exponential

A 554
B 650
C 725
D 750

$y = 2.46(1.21)^{30}$
$y = 749.02483...$

6. The table shows the number of students enrolled in the Algebra/Trig program at a high school the first 5 years the course was offered.

Year (x)	No. of students (y)
1	55
2	71
3	84
4	97
5	108

Which of the following equations most closely describes the relationship between the number of students enrolled and the number of years the class has existed?

linear...

A $y = x + 13$
B $y = 13x + 43$
C $y = 10x + 23$
D $y = 13x - 43$

$y = 13.2x + 43.4$

SOL AII.10 The student will identify, create, and solve real-world problems involving inverse variation, joint variation, and a combination of direct and inverse variations.

Key

Hints and Notes

Direct Variation	*Inverse Variation*	*Joint Variation*
$y = kx$ or $k = \frac{y}{x}$, where k is the constant of variation	$y = \frac{k}{x}$ or $k = xy$, where k is the constant of variation	$xyz = k$ or $y = \frac{k}{xz}$, where k is the constant of variation

PRACTICE AII.10

1. **The time it takes to travel a given distance varies inversely as the average rate of travel. Averaging 42 miles per hour, it takes John 5 hours to drive to Pittsburgh. If it took him 4 hours and 20 minutes to reach Pittsburgh on his last trip, what was his average rate of travel?**

   A 36.4 mph
   B ~~46.7 mph~~
   C 48.5 mph
   D 49.4 mph

   $t = \frac{k}{r}$ $5 = \frac{k}{42}$ $k = 210$

   $t = \frac{210}{r}$ $\therefore 4\frac{1}{3} = \frac{210}{r}$

   $r = 210 \div \frac{13}{3}$

2. **The volume (V) of a sphere varies directly with the cube of its radius (r). If k is the constant of proportionality, which is the formula for this relationship?**

   A $V = kr$
   B $V = kr^3$
   C $V = \frac{k}{r^3}$
   D $r = kV^3$

   $V = kr^3$

3. **Hooke's law states that the force required to stretch a spring varies directly with the distance the spring is stretched. If a 10 pound force stretches a spring 2 inches, what force is required to stretch 5 inches?**

   A 15 pounds
   B ~~20 pounds~~
   C 25 pounds
   D 30 pounds

   $F = kd$ $10 = k2$ $5 = k$

   $F = 5d$ $F = 5(5)$

4. **The amount of interest (I) owed on a loan varies directly with the length of time (t) of the loan. If k is the constant of proportionality, which formula represents this relationship?**

   A $I = kt$
   B $I = \frac{k}{t}$
   C $t = kI$
   D $t = \frac{k^2}{I}$

   $I = kt$

5. **Boyle's Law states that, for a fixed amount of gas, the volume of the gas at a constant temperature is inversely proportional to the pressure. If a certain gas occupies 9.84 liters at a pressure of 50 centimeters of mercury (cm Hg), what is the approximate pressure when the volume is increased to 12 liters?**

   A ~~39.8 cm Hg~~
   B 41.0 cm Hg
   C ~~43.2 cm Hg~~
   D 45.1 cm Hg

   $V = \frac{k}{P}$ $9.84 = \frac{k}{50}$ $492 = k$

   $V = \frac{492}{P}$ $12 = \frac{492}{P}$ $P = 492 \div 12$

6. **In which of the following is z directly proportional to x and inversely proportional to the square of y?**

   A $z = k\frac{x^2}{y}$
   B $z = kxy^2$
   C $z = \frac{x}{y^2}k$
   D $z = k\frac{y}{x}$

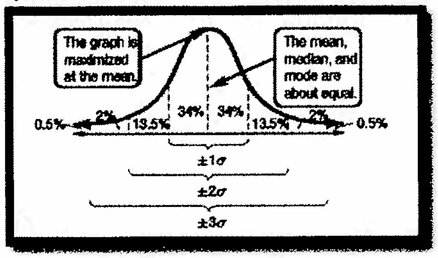
   $z = \frac{kx}{y^2}$

SOL AII.11 **The student will identify properties of a normal distribution and apply those properties to determine probabilities associated with areas under the standard normal curve.**

Key

Hints and Notes

- **The *Empirical Rule* for a Normal Distribution**



- ***Standard Deviation***: (s) describes how closely a set of data clusters about the mean.
- ***Z-score***: the measure of how many standard deviations an element falls above or below the mean of a set of data.

To find *mean, standard deviation, median,* etc. for a list of data using a graphics calculator:

- Enter data into L_1 (STAT>edit > L_1),
- then STAT> CALC>1-var Stats > ENTER > ENTER

To find a probability using a graphics calculator:

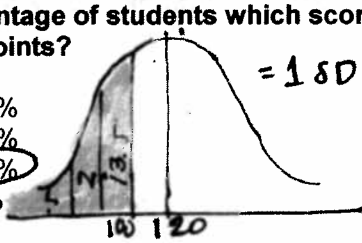
- Type 2nd > VARS > normalcdf (L, U, M, S) > ENTER

Lower limit of the random variable,
Upper limit of the random variable,
Mean of the distribution,
Standard deviation of the Distribution
Hint: use -99999999 for $-\infty$ and 99999999 $+\infty$

PRACTICE AII.11

1. **At Thomas Nelson Community College, the Pre-Test for Mathematics has 200 points on the test. The mean is 120 and the standard deviation is 20. What is the approximate percentage of students which score below 100 points?**

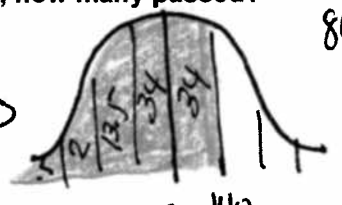
   A 50%
   B 34%
   C 16%
   D 7%



= 1 SD
$\sigma = 20$
USE the EMPRule!

2. **At Thomas Nelson Community College, the Pre-Test for Mathematics has 200 points on the test. The mean is 120 and the standard deviation is 20. In order to pass the Mathematics Pre-Test, a student must score 140 points. If 360 freshmen took the pre-test last fall, how many passed?**

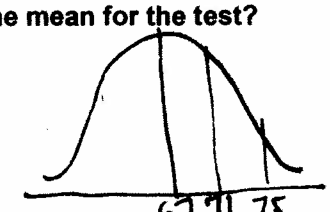
   A 200
   B 240
   C 302
   D 324



84% of 360
302.4

3. **On his midterm exam, Jimmy scored 75 points, which was exactly 2 standard deviations above the mean. If the standard deviation for the test is 4, what is the mean for the test?**

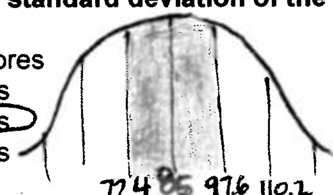
   A 79
   B 75
   C 71
   D 67



$\sigma = 4$
75−8

4. **Susie's test grades in Algebra II for the third quarter are 100, 80, 60, 88, 92, and 90. How many scores are within one standard deviation of the mean?**

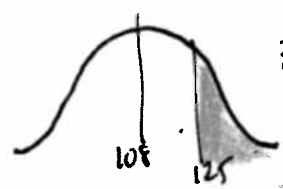
   A all 6 scores
   B 5 scores
   C 4 scores
   D 3 scores



$\sigma = 12.6$
$\mu = 85$
(Did Stat Calc → 1VAR STAT)

5. **A normally distributed set of 968 values has a standard deviation of 11 and a mean of 108. Which is closest to the number of values expected to be above 125?**

   A 910
   B 989
   C 210
   D 59



$\sigma = 11$
$\mu = 108$
$z = \frac{125-108}{11} = 1.545 = 1.55$
Area = .9394
1 − .9394 = .0606
.0606(968)
58.66

TEI (Technology Enhanced Item): Free-Response - For free-response questions, type your answer in the box. Be sure your answer is in appropriate form - simplest fraction, decimal, etc.

6. **Roanoke, Virginia had the folowing amounts of snowfall last January:**

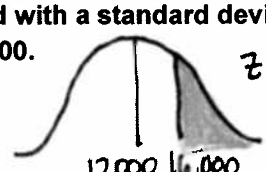
   4.2" 2.3" 6" 7.8" 10" 5.5" 12.5" 0.8"

   Find the mean and the standard deviation.

   Mean: $\boxed{6.14}$ (6.1375)

   Standard Deviation: $\boxed{3.63}$ (3.6324)

7. **A survey of 20 colleges found that seniors graduated with an average $12,000 in debt from student loans. The debt was normally distributed with a standard deviation of $3200. Find the probability that a senior graduated owing more than $16,000.**



$z = \frac{16000-12000}{3200}$
$z = 1.25$
Area: .8944
1 − .8944 = .1056

$\boxed{10.56\%}$

SOL AII.12 The student will compute and distinguish between permutations and combinations and use technology for applications.

Key

Hints and Notes

Permutation $_nP_r$	Combination $_nC_r$
• An ordering of ***n*** objects ***r*** at a time • where the **ORDER** of the objects **MATTERS** • **CALCULATOR:** MATH > PRB > $_nP_r$	• Selection of ***r*** objects taken from a group of ***n*** objects • where the **ORDER does NOT matter** • **CALCULATOR:** MATH > PRB > $_nC_r$

PRACTICE AII.12

1. If the digits cannot be repeated, how many 3-digit numbers can be formed using the digits 1, 2, 3, and 4?

dependent and order matters

A 24
B 48
C 64
D 256

$_4P_3$

2. Meredith has 14 girls on her softball team. She wants to have 2 co-captains. How many different choices of pairs of co-captains does she have?

A $_{14}P_2$
B 14!
C $_{14}C_2$
D 14

3. If numbers and letters can be repeated, how many different 6-digit license plates can be made if the first two positions are letters and the last four are digits?

independent events, ∴ use the Fundamental Counting Principle

A 492,804
B 676,000
C 6,760,000
D 455,625

$\overline{26} \cdot \overline{26} \cdot \overline{10} \cdot \overline{10} \cdot \overline{10} \cdot \overline{10}$

TEI (Technology Enhanced Item): Free-Response - For free-response questions, type your answer in the box. Be sure your answer is in appropriate form - simplest fraction, decimal, etc.

4. A committee of 3 teachers and 3 students is to be formed to judge a contest. If there are 7 students and 5 teachers to choose from, how many different committees could be formed? Type your answer in the box provided.

Teachers   Students

$_5C_3$ $_7C_3$

$10 \cdot 35 = 350$

$\boxed{350}$