

# 7-6 Rational Exponents

**Intro:**  $\left(b^{\frac{1}{2}}\right)^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}} = b^{\frac{1}{2} + \frac{1}{2}} = b^1 = b$

**Conclusion:** Since  $b^{\frac{1}{2}}$  is a number with a square = b, then  $b^{\frac{1}{2}} = \sqrt{b}$

**FORMULAS:**

$$b^{\frac{1}{n}} = \sqrt[n]{b^1} = \sqrt[n]{b}$$

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

**REWRITING AN EXPRESSION USING THE TWO TYPES OF NOTATIONS:**

**Rational Exponent Notation:**

**Example:**  $(\sqrt[3]{2})^5$  OR  $\sqrt[3]{2^5} = 2^{\frac{5}{3}}$

**Radical Notation:**

**Example:**  $45^{\frac{3}{2}} = (\sqrt[2]{45})^3 = \sqrt[2]{45^3}$

Express using rational exponents.

1.  $\sqrt[5]{x^2}$

2.  $\sqrt[3]{5}$

3.  $\sqrt[6]{16}$

4.  $\sqrt[10]{x^8}$

5.  $\sqrt{36x^5y^7}$

6.  $\sqrt[4]{16a^2b^3}$

7.  $\sqrt[3]{28x^2y^4z^3}$

8.  $3\sqrt[4]{8n^{11}w^2}$

Express in simplest radical form.

9.  $x^{\frac{4}{5}}$

10.  $8^{\frac{1}{6}}$

11.  $x^{\frac{1}{4}}$

12.  $\sqrt[10]{16}$

13.  $\sqrt[6]{25}$

14.  $2^{\frac{5}{4}}a^{\frac{3}{4}}y^{\frac{9}{4}}$

15.  $w^{\frac{3}{5}}n^{\frac{5}{3}}$

16.  $a^{\frac{2}{3}}b^{\frac{1}{8}}c^{\frac{1}{2}}$

**FINDING n<sup>th</sup> ROOTS:** Example:  $36^{\frac{3}{2}} = (\sqrt[2]{36})^3 = 6^3 = 216$

Hint: take the root first, and then raise it to the power.

Evaluate each of the following without using a calculator.

17.  $125^{\frac{1}{3}}$

18.  $16^{-\frac{1}{2}}$

19.  $(\sqrt[3]{-27})^2$

20.  $(-8)^{\frac{5}{3}}$

21.  $9^{-\frac{3}{2}}$

# Properties of Rational Exponents

The laws of exponents also apply to rational exponents. The following summarizes those properties:

<u>PROPERTY</u>	<u>EXAMPLE</u>
1. $a^m \cdot a^n = a^{m+n}$	$6^{\frac{1}{2}} \cdot 6^{\frac{1}{3}} = 6^{\frac{1}{2} + \frac{1}{3}} = 6^{\frac{3}{6} + \frac{2}{6}} = 6^{\frac{5}{6}}$
2. $(a^m)^n = a^{mn}$	$\left(\frac{1}{5^3}\right)^6 = \frac{1}{5^{3 \cdot 6}} = \frac{1}{5^{18}}$
3. $(ab)^m = a^m \cdot b^m$	$\left(27^{\frac{1}{3}} \cdot 6^{\frac{1}{4}}\right)^2 = 27^{\frac{2}{3}} \cdot 6^{\frac{2}{4}} = (3^3)^{\frac{2}{3}} \cdot 6^{\frac{1}{2}} = 3^2 \cdot 6^{\frac{1}{2}} = 9 \cdot 6^{\frac{1}{2}}$
4. $a^{-m} = \frac{1}{a^m}, a \neq 0$	$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{6}$
5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$	$\frac{6}{6^4} = 6^{1-\frac{3}{4}} = 6^{\frac{1}{4}}$
6. $\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}, b \neq 0$	$\left(\frac{18^4}{9^4}\right)^3 = \left(\frac{(9 \cdot 2)^4}{9^4}\right)^3 = \left(\frac{9^4 \cdot 2^4}{9^4}\right)^3 = \left(2^4\right)^3 = 2^{12}$

Simplify each expression.

22.  $x^{\frac{3}{4}} \cdot x^{\frac{1}{4}}$

23.  $a^{\frac{1}{6}} \cdot a^{\frac{1}{3}}$

24.  $5^{\frac{1}{4}} \cdot 5^{\frac{2}{3}}$

25.  $7^{\frac{1}{8}} \cdot 7^{\frac{3}{4}}$

26.  $\left(5^{\frac{2}{3}}\right)^{\frac{1}{2}}$

27.  $\left(x^{\frac{1}{3}}\right)^{12}$

28.  $\frac{b^{\frac{11}{5}}}{b^{\frac{7}{5}}}$

29.  $\frac{4^{\frac{15}{7}}}{4^7}$

30.  $\left(15^{\frac{3}{5}}\right)^{\frac{2}{7}}$

31.  $\left(n^{\frac{2}{5}}\right)^{10}$

32.  $(625j^8k^4m^6)^{\frac{1}{4}}$

33.  $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$