

7-6 Rational Exponents

Intro: $(b^{\frac{1}{2}})^2 = b^{\frac{1}{2}} \cdot b^{\frac{1}{2}} = b^{\frac{1}{2} + \frac{1}{2}} = b^1 = b$

Conclusion: Since $b^{\frac{1}{2}}$ is a number with a square = b , then $b^{\frac{1}{2}} = \sqrt{b}$

FORMULAS:

$$b^{\frac{1}{n}} = \sqrt[n]{b^1} = \sqrt[n]{b}$$

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \sqrt[n]{b^m} = (\sqrt[n]{b})^m$$

REWRITING AN EXPRESSION USING THE TWO TYPES OF NOTATIONS:

Rational Exponent Notation:

Example: $(\sqrt[3]{2})^5$ OR $\sqrt[3]{2^5} = 2^{\frac{5}{3}}$

Radical Notation:

Example: $45^{\frac{3}{2}} = (\sqrt[2]{45})^3 = \sqrt[2]{45^3}$

Express using rational exponents.

1. $\sqrt[5]{x^2} \rightarrow x^{\frac{2}{5}}$

2. $\sqrt[3]{5} \rightarrow 5^{\frac{1}{3}}$

3. $\sqrt[6]{16} \rightarrow 16^{\frac{1}{6}} \rightarrow \sqrt[6]{2^4} = 2^{\frac{4}{6}} = 2^{\frac{2}{3}}$

4. $\sqrt[10]{x^8} \rightarrow x^{\frac{8}{10}} = x^{\frac{4}{5}}$

5. $\sqrt{36x^5y^7} \rightarrow 6x^{\frac{5}{2}}y^{\frac{7}{2}}$

6. $\sqrt[4]{16a^2b^3} \rightarrow 2a^{\frac{1}{2}}b^{\frac{3}{4}}$

7. $\sqrt[3]{28x^2y^4z^3} \rightarrow 28^{\frac{1}{3}}x^{\frac{2}{3}}y^{\frac{4}{3}}z$

8. $\sqrt[3]{8n^{11}w^2} \rightarrow 2 \cdot 2^{\frac{2}{3}}n^{\frac{11}{3}}w^{\frac{2}{3}}$

Express in simplest radical form.

9. $x^{\frac{4}{5}} \rightarrow \sqrt[5]{x^4}$

10. $8^{\frac{1}{6}} \rightarrow \sqrt[6]{8} \rightarrow \sqrt[6]{2^3} = 2^{\frac{1}{2}} = \sqrt{2}$

11. $7^{\frac{1}{4}} \rightarrow \sqrt[4]{7}$

12. $\sqrt[10]{16} \rightarrow \sqrt[10]{2^4} \rightarrow 2^{\frac{4}{10}} = 2^{\frac{2}{5}} = \sqrt[5]{4}$

13. $\sqrt[6]{25} \rightarrow \sqrt[6]{5^2} \rightarrow 5^{\frac{2}{6}} = 5^{\frac{1}{3}} = \sqrt[3]{5}$

14. $2^{\frac{5}{4}}a^{\frac{3}{4}}y^{\frac{9}{4}} \rightarrow \sqrt[4]{2^5a^3y^9} \rightarrow 2y^2\sqrt[4]{2a^3y}$

15. $w^{\frac{3}{5}}n^{\frac{5}{3}} \rightarrow w^{\frac{9}{15}}n^{\frac{25}{15}} \rightarrow \sqrt[15]{w^9n^{25}} \rightarrow n\sqrt[15]{w^9n^{10}}$

16. $a^{\frac{2}{3}}b^{\frac{1}{8}}c^{\frac{1}{2}} \rightarrow a^{\frac{16}{24}}b^{\frac{3}{24}}c^{\frac{12}{24}} \rightarrow \sqrt[24]{a^{16}b^3c^{12}}$

FINDING n^{th} ROOTS: Example: $36^{\frac{3}{2}} = (\sqrt{36})^3 = 6^3 = 216$

Hint: take the root first, and then raise it to the power.

Evaluate each of the following without using a calculator.

17. $125^{\frac{1}{3}} \rightarrow 5$

18. $16^{-\frac{1}{2}} \rightarrow \frac{1}{\sqrt{16}} = \frac{1}{4}$

19. $(\sqrt[3]{-27})^2 \rightarrow (-3)^2 = 9$

20. $(-8)^{\frac{5}{3}} \rightarrow (-2)^5 = -32$

21. $9^{-\frac{3}{2}} \rightarrow \frac{1}{\sqrt{9^3}} = \frac{1}{27}$

Properties of Rational Exponents

The laws of exponents also apply to rational exponents. The following summarizes those properties:

PROPERTY

EXAMPLE

1. $a^m \cdot a^n = a^{m+n}$

$$6^{\frac{1}{2}} \cdot 6^{\frac{1}{3}} = 6^{\frac{1}{2} + \frac{1}{3}} = 6^{\frac{3}{6} + \frac{2}{6}} = 6^{\frac{5}{6}}$$

2. $(a^m)^n = a^{mn}$

$$\left(5^{\frac{1}{3}}\right)^6 = 5^{\frac{6}{3}} = 5^2 = 25$$

3. $(ab)^m = a^m \cdot b^m$

$$\left(27^{\frac{1}{3}} \cdot 64^{\frac{1}{4}}\right)^2 = 27^{\frac{2}{3}} \cdot 64^{\frac{2}{4}} = (3^3)^{\frac{2}{3}} \cdot 6 \cdot 2 = 3^2 \cdot 3 \cdot 2 = 3^3 \cdot 2 = 27 \cdot 2 = 54$$

4. $a^{-m} = \frac{1}{a^m}, a \neq 0$

$$36^{-\frac{1}{2}} = \frac{1}{36^{\frac{1}{2}}} = \frac{1}{6}$$

5. $\frac{a^m}{a^n} = a^{m-n}, a \neq 0$

$$\frac{6^{\frac{1}{3}}}{6^{\frac{1}{4}}} = 6^{\frac{1}{3} - \frac{1}{4}} = 6^{\frac{4}{12} - \frac{3}{12}} = 6^{\frac{1}{12}}$$

6. $\frac{a^m}{b^m} = \left(\frac{a}{b}\right)^m, b \neq 0$

$$\left(\frac{18^4}{9^4}\right)^3 = \left(\frac{(9 \cdot 2)^4}{9^4}\right)^3 = \left(\frac{9^4 \cdot 2^4}{9^4}\right)^3 = \left(\frac{16}{1}\right)^3 = 16^3 = 4096$$

Simplify each expression.

22. $x^{\frac{3}{4}} \cdot x^{\frac{1}{4}}$

$$x^{\frac{4}{4}} = x$$

23. $a^{\frac{1}{6}} \cdot a^{\frac{1}{3}}$

$$a^{\frac{1}{6} + \frac{2}{6}} = a^{\frac{3}{6}} = a^{\frac{1}{2}} = \sqrt{a}$$

24. $5^{\frac{1}{4}} \cdot 5^{\frac{2}{3}}$

$$5^{\frac{3}{12} + \frac{8}{12}} = 5^{\frac{11}{12}} = \sqrt[12]{5^{11}}$$

25. $7^{\frac{1}{8}} \cdot 7^{\frac{3}{4}}$

$$7^{\frac{1}{8} + \frac{6}{8}} = 7^{\frac{7}{8}} = \sqrt[8]{7^7}$$

26. $\left(5^{\frac{2}{3}}\right)^{\frac{1}{2}}$

$$5^{\frac{1}{3}} = \sqrt[3]{5}$$

27. $\left(x^{\frac{1}{3}}\right)^{12}$

$$x^4$$

28. $\frac{b^{\frac{11}{5}}}{b^{\frac{7}{5}}}$

$$b^{\frac{4}{5}} = \sqrt[5]{b^4}$$

29. $\frac{4^{\frac{15}{7}}}{4^{\frac{1}{7}}}$

$$4^{\frac{14}{7}} = 4^2 = 16$$

30. $\left(15^{\frac{3}{5}}\right)^{\frac{2}{7}}$

$$15^{\frac{6}{35}} = \sqrt[35]{15^6}$$

31. $\left(n^{\frac{2}{5}}\right)^{10^2}$

$$n^4$$

32. $(625j^8k^4m^6)^{\frac{1}{4}}$

$$5j^2km^{\frac{3}{2}} = 5j^2k\sqrt{m^3}$$

33. $\frac{x^{\frac{2}{3}}}{x^{\frac{1}{4}}}$

$$x^{\frac{5}{12}} = \sqrt[12]{x^5}$$