6-5 Solving Polynomials by Factoring

FACTORING ORDER OF OPERATIONS

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FACTOR OUT THE GCF

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LOOK FOR A PATTERN!

Master

BINOMIALS

Difference of Squares
$a^2 - b^2 = (a + b)(a - b)$

Difference/Sum of Cubes
$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

TRINOMIALS

Trinomial Squares
$a^2 + 2ab + b^2 = (a + b)^2$
$a^2 - 2ab + b^2 = (a - b)^2$

$x^2 + bx + c$
Find 2 #'s whose sum is b
and product is c

$ax^2 + bx + c$
Find 2 #'s whose sum is b
and product is a • c, and factor
using the slip-slide method

OTHER

Factor by Grouping
Use when there are 4 or more terms

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Look at the terms in pairs $\quad x^2 - 2xy + x - 2y \quad = (x^2 - 2xy) + (x - 2y)$
Factor out the GCF of each pair $\qquad\qquad\qquad = x(x - 2y) + 1(x - 2y)$
$(x - 2y)$ is the new GCF of the 2 new terms! $\quad = (x - 2y)(x + 1)$

Practice factoring the sum or difference of cubes.

1. $x^3 + 64$
$(x+4)(x^2-4x+16)$

2. $x^3 - 343$
$(x-7)(x^2+7x+49)$

3. $8m^3 - 1$
$(2m-1)(4m^2+2m+1)$

4. $27x^3 + 1$
$(3x+1)(9x^2-3x+1)$

5. $64x^3 + 27$
$(4x+3)(16x^2-2x+9)$

6. $4x^6 + 108y^3$
$4(x^6+27y^3)$
$4(x^2+3y)(x^4+3x^2y+9y^2)$

Practice factoring by grouping.

7. $6xy + 8x - 21y - 28$
$2x(3y+4) - 7(3y+4)$
$(3y+4)(2x-7)$

8. $3xy - 21y + 5x - 35$
$3y(x-7) + 5(x-7)$
$(x-7)(3y+5)$

9. $8m^2n - 5m - 24mn + 15$
$m(8mn-5) - 3(8mn-5)$
$(8mn-5)(m-3)$

Practice factoring the difference of two squares.

10. $16x^4 - 81$
$(4x^2+9)(4x^2-9)$
$(4x^2+9)(2x+3)(2x-3)$

11. $10x^2 - 40$
$10(x^2-4)$
$10(x+2)(x-2)$

12. $x^4 - 625$
$(x^2+25)(x^2-25)$
$(x^2+25)(x+5)(x-5)$

Practice factoring higher degree trinomials.

13. $x^5 + 4x^4 - 32x^3$
$x^3(x^2+4x-32)$
$x^3(x+8)(x-4)$

14. $x^4 + x^2 - 6$
$(x^2+3)(x^2-2)$

15. $x^4 - 8x^2 - 9$
$(x^2-9)(x^2+1)$
$(x+3)(x-3)(x^2+1)$

Solve each polynomial function: Factor first, then set each factor equal to zero and solve completely.

16. $x^4 + x^2 - 6 = 0$

$(x^2+3)(x^2-2)=0$

$x^2+3=0 \quad x^2-2=0$

$x^2=-3 \quad x^2=2$

$\boxed{x=\pm i\sqrt{3} \quad x=\pm\sqrt{2}}$

17. $3x^4 - 3 = 0$

$3(x^4-1)$

$3(x^2+1)(x^2-1)$

$3(x^2+1)(x+1)(x-1)=0$

$x^2+1=0$

$x^2=-1$

$\boxed{x=\pm i, -1, 1}$

18. $2x^4 - 200x^2 = 0$

$2x^2(x^2-100)$

$2x^2(x+10)(x-10)$

$\boxed{x=0,0,\pm 10}$

19. $x^3 + 6x^2 - 4x - 24 = 0$

$x^2(x+6)-4(x+6)$

$(x+6)(x^2-4)=0$

$(x+6)(x+2)(x-2)=0$

$\boxed{x=-6,-2,+2}$

20. $3x^3 - x^2 + 3x - 1 = 0$

$x^2(3x-1)+1(3x-1)$

$(3x-1)(x^2+1)=0$

$3x-1=0 \quad x^2+1=0$

$3x=1 \quad x^2=-1$

$\boxed{x=\frac{1}{3}, \pm i}$

21. $x^4 - 10x^2 + 9 = 0$

$(x^2-9)(x^2-1)=0$

$(x+3)(x-3)(x+1)(x-1)=0$

$\boxed{x=\pm 3, \pm 1}$

22. $x^3 - 8 = 0$

$(x-2)(x^2+2x+4)$

$x=2 \qquad \dfrac{-2\pm\sqrt{4-4(4)}}{2}$

$\dfrac{-2\pm\sqrt{-12}}{2}$

$\boxed{x=2, -1\pm i\sqrt{3}}$ $\dfrac{-2\pm 2i\sqrt{3}}{2}$

23. $x^4 - x^2 - 12 = 0$

$(x^2-4)(x^2+3)=0$

$(x+2)(x-2)(x^2+3)=0$

$\boxed{x=\pm 2, \pm i\sqrt{3}}$

24. $8x^3 + 1000 = 0$

$8(x^3+125)=0$

$8(x+5)(x^2-5x+25)$

$x=-5 \qquad \dfrac{5\pm\sqrt{25-4(25)}}{2}$

$\boxed{x=-5, \dfrac{5\pm 5i\sqrt{3}}{2}}$ $\dfrac{5\pm\sqrt{-75}}{2}$

$\dfrac{5\pm 5i\sqrt{3}}{2}$

25. $27x^4 - 3x^2 = 0$

$3x^2(9x^2-1)=0$

$3x^2(3x+1)(3x-1)=0$

$\boxed{x=0,0,-\frac{1}{3},\frac{1}{3}}$

26. $x^4 - 4x^2 - 5 = 0$

$(x^2-5)(x^2+1)=0$

$\boxed{x=\pm\sqrt{5}, \pm i}$

27. $121x^2 - 49 = 0$

$(11x+7)(11x-7)=0$

$\boxed{x=\pm\frac{7}{11}}$