7-4 & 7-5 nth Roots & Operations with Radical Expressions

Master

Simplify Radicals: Finding the square root of a number and squaring a number are inverse operations. Also, the inverse of raising a number to the nth power is finding the nth root of a number.

The n^th Root of a Number: To solve a square root you must find the number that, when multiplied by itself, results in the radicand. To solve an n^th root you must find the number that, when multiplied by itself *n* times, results in the radicand.

index → $\sqrt[n]{a}$ ← radical sign; radicand

Note: When there is no index shown, the index is understood to be 2 (square root)

Suppose n is an integer greater than 1 and a is a real number. The following will be true.

a	If n is even:	If n is odd:
a > 0	There are 2 real roots: $\sqrt[4]{16} = \pm 2$	There is one real positive root: $\sqrt[5]{32} = 2$
a < 0	There are no real roots: $\sqrt[4]{-16}$ = no real roots	There is one real negative root: $\sqrt[3]{-64} = -4$
a = 0	This is one root: $\sqrt[n]{0} = 0$	This is one root: $\sqrt[n]{0} = 0$

Principal Root: Some numbers have more than one real nth root. For example, 64 has 2 square roots, ±8, since 8^2 and $(-8)^2$ both equal 64. When n is even, there is more than one real root; the positive root is called the **principal root**.

Formulas: $\sqrt[n]{a^n} = a$ $\quad \sqrt[n]{a^{2n}} = a^2$ $\quad \sqrt[n]{a^{3n}} = a^3$ $\quad \sqrt[n]{a^m} = a^{\frac{m}{n}}$ $\quad \sqrt[n]{(a+b)^n} = a+b$

1-14: Simplify each expression. Pay attention to the details... signs DO matter!

1. $\sqrt{36}$ 6	2. $\pm\sqrt{36}$ ±6	3. $-\sqrt{36}$ −6	4. $\sqrt{-36}$ ±6i	5. $-\sqrt{-36}$ ±6i	6. $\sqrt[3]{64}$ 4	7. $-\sqrt[3]{64}$ −4
8. $\sqrt[3]{-64}$ −4	9. $-\sqrt[3]{-64}$ 4	10. $\sqrt[4]{81}$ 3	11. $-\sqrt[4]{81}$ −3	12. $\sqrt[4]{-81}$ imag. sol.	13. $\sqrt[4]{x^8}$ x^2	14. $\sqrt[3]{x^{12}}$ x^4

Approximate n^th roots with a calculator: Example: $\sqrt[5]{32}$

Option 1: Use the MATH menu and the $\sqrt[x]{\ }$ function. First input *n*, then $\sqrt[x]{\ }$, then the radicand, then ENTER. (Keystrokes: 5 → MATH → $\sqrt[x]{\ }$ → 32 → ENTER = 2)

Option 2: Enter 32^(1 ÷ 5) → ENTER

15-18: Use a calculator to approximate each value to three decimal places.

15. $\sqrt{1050}$ 32.404	16. $\sqrt[3]{-15}$ −2.466	17. $\sqrt[5]{100}$ 2.5119	18. $-\sqrt[4]{500}$ −4.729

19-22: Write each expression in simplified radical form.

19. $\sqrt{x^2 - 4x + 4}$ $\sqrt{(x-2)^2}$ $x-2$	20. $\sqrt[3]{-(y-9)^9}$ $-(y-9)^{\frac{9}{3}}$ $-(y-9)^3$	21. $\sqrt[8]{x^{16}y^8}$ $\sqrt[8]{x^{32}y^{16}}$ $x^{\frac{32}{8}} y^{\frac{16}{8}}$ $x^4 y^2$	22. $\pm\sqrt{49x^4}$ $\pm 7x^2$

Simplest Form of a Radical: For a radical to be in simplest form, it must meet the following conditions:

- The index n is as small as possible
- The radicand contains no factors other than 1 that are nth powers of an integer or polynomial
- The radicand contains no fractions
- No radicals appear in a denominator

23-26: Simplify each radical completely.

23. $\sqrt{12c^6d^3}$ $\sqrt{4\cdot 3c^6d^2d}$ $2c^3d\sqrt{3d}$	**24.** $\sqrt[3]{27y^{12}z^7}$ $3\sqrt[3]{y^{12}z^6z}$ $3y^4z^2\sqrt[3]{z}$	**25.** $\sqrt{50a^5b^9}$ $\sqrt{25\cdot 2a^4ab^8b}$ $5a^2b^4\sqrt{2ab}$	**26.** $\sqrt[3]{250m^{30}p^{20}}$ $\sqrt[3]{125\cdot 2m^{30}p^{18}p^2}$ $5m^{10}p^6\sqrt[3]{2p^2}$

Multiplying Radicals: You must use the Product Property.

PRODUCT PROPERTY: $\sqrt[n]{a\bullet b}=\sqrt[n]{a}\bullet\sqrt[n]{b}$ *n can be odd or even (a & b must be positive)*	**Examples: a.** a. $\sqrt[3]{25}\bullet\sqrt[3]{5}=\sqrt[3]{25\bullet 5}=\sqrt[3]{125}=5$ b. $\sqrt[4]{64}=\sqrt[4]{16\bullet 4}=\sqrt[4]{16}\bullet\sqrt[4]{4}=2\sqrt[4]{4}$

27-28: Simplify each radical completely.

27. $6\sqrt{8c^3d^5}\bullet 4\sqrt{2cd^3}$ $24\sqrt{16c^4d^8}$ $24\cdot 4\cdot c^2\cdot d^4$ $\quad 96c^2d^4$	**28.** $5\sqrt[3]{100a^2}\bullet\sqrt[3]{10a}$ $5\sqrt[3]{1000a^3}$ $5\cdot 10a = 50a$

Dividing Radicals: You must use the Quotient Property or Rationalize the Denominator when it doesn't reduce.

QUOTIENT PROPERTY: $\sqrt[n]{\frac{a}{b}}=\frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ *n must be positive and b can't equal zero*	**Examples:** a. $\frac{\sqrt[3]{32}}{\sqrt[3]{4}}=\sqrt[3]{\frac{32}{4}}=\sqrt[3]{8}=2$ b. $\sqrt[4]{\frac{7}{8}}=\frac{\sqrt[4]{7}}{\sqrt[4]{8}}=\frac{\sqrt[4]{7}}{\sqrt[4]{2^3}}\bullet\frac{\sqrt[4]{2}}{\sqrt[4]{2}}=\frac{\sqrt[4]{7\bullet 2}}{\sqrt[4]{2^4}}=\frac{\sqrt[4]{14}}{2}$

How to Rationalize the Denominator

If the denominator is:	Multiply the numerator and denominator by:	Examples:
$\sqrt{b}$	$\sqrt{b}$	$\frac{3}{\sqrt{2}}=\frac{3}{\sqrt{2}}\bullet\frac{\sqrt{2}}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{6}{\sqrt[4]{2}}=\frac{6}{\sqrt[4]{2}}\bullet\frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}}=\frac{6\sqrt[4]{8}}{2}=3\sqrt[4]{8}$
$a\sqrt{b}+c\sqrt{d}$ Or $a\sqrt{b}-c\sqrt{d}$	$a\sqrt{b}-c\sqrt{d}$ Or $a\sqrt{b}+c\sqrt{d}$	$\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}}=\frac{\sqrt{2}}{(\sqrt{5}-\sqrt{3})}\bullet\frac{(\sqrt{5}+\sqrt{3})}{(\sqrt{5}+\sqrt{3})}=\frac{\sqrt{10}+\sqrt{6}}{5+\sqrt{15}-\sqrt{15}-3}=\frac{\sqrt{10}+\sqrt{6}}{2}$

29-32: Simplify each radical completely.

29. $\frac{\sqrt{15}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{30}}{2}$	**30.** $\sqrt{\frac{y^8}{x^7}}\cdot\frac{\sqrt{x}}{\sqrt{x}}$ $\quad\frac{\sqrt{xy^8}}{\sqrt{x^8}}$ $\frac{y^4\sqrt{x}}{x^4}$	**31.** $\sqrt[3]{\frac{2}{9x}}=\frac{\sqrt[3]{2}}{\sqrt[3]{3^2x}}\cdot\frac{\sqrt[3]{3x^2}}{\sqrt[3]{3x^2}}$ $\frac{\sqrt[3]{6x^2}}{\sqrt[3]{3^3x^3}}$ $\quad\frac{\sqrt[3]{6x^2}}{3x}$	**32.** $\sqrt[4]{\frac{3}{4y}}$ $\quad\frac{\sqrt[4]{3}}{\sqrt[4]{2^2y}}\cdot\frac{\sqrt[4]{2^2y^3}}{\sqrt[4]{2^2y^3}}=\frac{\sqrt[4]{12y^3}}{\sqrt[4]{2^4y^4}}$ $\frac{\sqrt[4]{12y^3}}{2y}$

Adding & Subtracting Radicals: You must have like terms to add or subtract radicals!

Two radicals are like radicals if they have the same index **and** the same radicand. Use the distributive property t add or subtract like radicals	**Example:** $5\sqrt[4]{x^3}-3\sqrt[4]{x^3}=(5-3)\sqrt[4]{x^3}=2\sqrt[4]{x^3}$

33-34: Simplify each radical completely.

33. $3\sqrt{45}-5\sqrt{80}+4\sqrt{20}$ $3\sqrt{9}\sqrt{5}-5\sqrt{16}\sqrt{5}+4\sqrt{4}\sqrt{5}$ $3\cdot 3\sqrt{5}-5\cdot 4\sqrt{5}+4\cdot 2\sqrt{5}$ $9\sqrt{5}-20\sqrt{5}+8\sqrt{5}$	**34.** $9\sqrt[3]{16}+3\sqrt[3]{2}-\sqrt[3]{128}$ $9\sqrt[3]{8}\sqrt[3]{2}+3\sqrt[3]{2}-\sqrt[3]{64\cdot 2}$ $9\cdot 2\sqrt[3]{2}+3\sqrt[3]{2}-4\sqrt[3]{2}$ $18\sqrt[3]{2}+3\sqrt[3]{2}-4\sqrt[3]{2}$

$\sqrt{5}(9-20+8)\sqrt{5}=-3\sqrt{5}$ $\qquad$ $\sqrt[3]{2}(18+3-4)=17\sqrt[3]{2}$