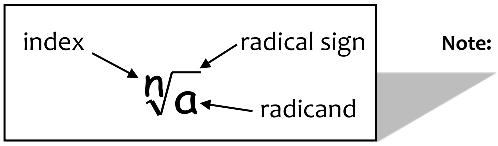
Day 05 Operations with Radical Expressions

Simplify Radicals: Finding the square root of a number and squaring a number are ________________ operations. Also, the inverse of raising a number to the nth power is finding the ______ root of a number.

The n^th Root of a Number: To solve a square root you must find the number that, when multiplied by itself, results in the radicand. To solve an n^th root you must find the number that, when multiplied by itself *n* times, results in the radicand.



Note: When there is no index shown, the index is understood to be 2 (square root)

Suppose n is an integer greater than 1 and a is a real number. The following will be true.

a	If n is even:	If n is odd:
a > 0	There are 2 real roots: $\sqrt[4]{16} = \pm 2$	There is one real positive root: $\sqrt[5]{32} = 2$
a < 0	There are no real roots: $\sqrt[4]{-16}$ = no real roots	There is one real negative root: $\sqrt[3]{-64} = -4$
a = 0	This is one root: $\sqrt[n]{0} = 0$	This is one root: $\sqrt[n]{0} = 0$

Principal Root: Some numbers have more than one real nth root. For example, 64 has 2 square roots, ±8, since 8^2 and $(-8)^2$ both equal 64. When n is even, there is more than one real root; the positive root is called the **principal root**.

Formulas: $\sqrt[n]{a^n}$ = ______ $\sqrt[n]{a^{2n}}$ = ______ $\sqrt[n]{a^{3n}}$ = ______ $\sqrt[n]{a^m}$ = ______ $\sqrt[n]{(a+b)^n}$ = ______

1-14: Simplify each expression. Pay attention to the details… signs DO matter!

1. $\sqrt{36}$	**2.** $\pm\sqrt{36}$	**3.** $-\sqrt{36}$	**4.** $\sqrt{-36}$	**5.** $-\sqrt{-36}$	**6.** $\sqrt[3]{64}$	**7.** $-\sqrt[3]{64}$
8. $\sqrt[3]{-64}$	**9.** $-\sqrt[3]{-64}$	**10.** $\sqrt[4]{81}$	**11.** $-\sqrt[4]{81}$	**12.** $\sqrt[4]{-81}$	**13.** $\sqrt[4]{x^8}$	**14.** $\sqrt[3]{x^{12}}$

Approximate n^th roots with a calculator: **Example:** $\sqrt[5]{32}$

Option 1: Use the MATH menu and the $\sqrt[x]{\ }$ function. First input *n*, then $\sqrt[x]{\ }$, then the radicand, then ENTER. (Keystrokes: 5 → MATH → $\sqrt[x]{\ }$ → 32 → ENTER = 2)

Option 2: Enter 32^ (1 ÷ 5) → ENTER

15-18: Use a calculator to approximate each value to three decimal places.

15. $\sqrt{1050}$	**16.** $\sqrt[3]{-15}$	**17.** $\sqrt[5]{100}$	**18.** $-\sqrt[4]{500}$

19-22: Write each expression in simplified radical form.

19. $\sqrt{x^2 - 4x + 4}$	**20.** $\sqrt[3]{-(y-9)^9}$	**21.** $\sqrt[8]{x^{16}y^8}$ $\sqrt[8]{x^{16}y^8}$	**22.** $\pm\sqrt{49x^4}$

Simplest Form of a Radical: **For a radical to be in simplest form, it must meet the following conditions:**

- The index n is as small as possible
- The radicand contains no factors other than 1 that are nth powers of an integer or polynomial
- The radicand contains no fractions
- No radicals appear in a denominator

23-26: Simplify each radical completely.

23. $\sqrt{12c^6d^3}$	**24.** $\sqrt[3]{27y^{12}z^7}$	**25.** $\sqrt{50a^5b^9}$	**26.** $\sqrt[3]{250m^{30}p^{20}}$

Multiplying Radicals: You must use the Product Property.

PRODUCT PROPERTY: $\sqrt[n]{a \bullet b} = \sqrt[n]{a} \bullet \sqrt[n]{b}$ *n can be odd or even (a & b must be positive)*	**Examples:** a. $\sqrt[3]{25} \bullet \sqrt[3]{5} = \sqrt[3]{25 \bullet 5} = \sqrt[3]{125} = 5$ b. $\sqrt[4]{64} = \sqrt[4]{16 \bullet 4} = \sqrt[4]{16} \bullet \sqrt[4]{4} = 2\sqrt[4]{4}$

27-28: Simplify each radical completely.

27. $6\sqrt{8c^3d^5} \bullet 4\sqrt{2cd^3}$	**28.** $5\sqrt[3]{100a^2} \bullet \sqrt[3]{10a}$

Dividing Radicals: You must use the Quotient Property or Rationalize the Denominator when it doesn't reduce.

QUOTIENT PROPERTY: $\sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ *n must be positive and b can't equal zero*	**Examples:** a. $\frac{\sqrt[3]{32}}{\sqrt[3]{4}} = \sqrt[3]{\frac{32}{4}} = \sqrt[3]{8} = 2$ b. $\sqrt[4]{\frac{7}{8}} = \frac{\sqrt[4]{7}}{\sqrt[4]{8}} = \frac{\sqrt[4]{7}}{\sqrt[4]{2^3}} \bullet \frac{\sqrt[4]{2}}{\sqrt[4]{2}} = \frac{\sqrt[4]{7 \bullet 2}}{\sqrt[4]{2^4}} = \frac{\sqrt[4]{14}}{2}$

How to Rationalize the Denominator

If the denominator is:	Multiply the numerator and denominator by:	Examples:
$\sqrt{b}$	$\sqrt{b}$	$\frac{3}{\sqrt{2}} = \frac{3}{\sqrt{2}} \bullet \frac{\sqrt{2}}{\sqrt{2}} = \frac{3\sqrt{2}}{2}$
$\sqrt[n]{b^x}$	$\sqrt[n]{b^{n-x}}$	$\frac{6}{\sqrt[4]{2}} = \frac{6}{\sqrt[4]{2}} \bullet \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \frac{6\sqrt[4]{8}}{2} = 3\sqrt[4]{8}$
$a\sqrt{b} + c\sqrt{d}$ Or $a\sqrt{b} - c\sqrt{d}$	$a\sqrt{b} - c\sqrt{d}$ Or $a\sqrt{b} + c\sqrt{d}$	$\frac{\sqrt{2}}{\sqrt{5}-\sqrt{3}} = \frac{\sqrt{2}}{\left(\sqrt{5}-\sqrt{3}\right)} \bullet \frac{\left(\sqrt{5}+\sqrt{3}\right)}{\left(\sqrt{5}+\sqrt{3}\right)} = \frac{\sqrt{10}+\sqrt{6}}{5+\sqrt{15}-\sqrt{15}-3} = \frac{\sqrt{10}+\sqrt{6}}{2}$

29-32: Simplify each radical completely.

29. $\frac{\sqrt{15}}{\sqrt{2}}$	**30.** $\sqrt{\frac{y^8}{x^7}}$	**31.** $\sqrt[3]{\frac{2}{9x}}$	**32.** $\sqrt[4]{\frac{3}{4y}}$

Adding & Subtracting Radicals: You must have like terms to add or subtract radicals!

Two radicals are like radicals if they have the same index **and** the same radicand. Use the distributive property t add or subtract like radicals	**Example:** $5\sqrt[4]{x^3} - 3\sqrt[4]{x^3} = (5-3)\sqrt[4]{x^3} = 2\sqrt[4]{x^3}$

33-34: Simplify each radical completely.

33. $3\sqrt{45} - 5\sqrt{80} + 4\sqrt{20}$	**34.** $9\sqrt[3]{16} + 3\sqrt[3]{2} - \sqrt[3]{128}$