

2-6 Algebraic Proof

ALGEBRAIC PROPERTIES OF EQUALITY FOR ANY REAL NUMBERS a, b, and c:	
Addition Property	If $a = b$, then $a + c = b + c$.
Subtraction Property	If $a = b$, then $a - c = b - c$.
Multiplication Property	If $a = b$, then $a \cdot c = b \cdot c$.
Division Property	If $a = b$ and $c \neq 0$, then $\frac{a}{c} = \frac{b}{c}$.
Reflexive Property	For any real number, $a = a$.
Symmetric Property	If $a = b$, then $b = a$.
Transitive Property	If $a = b$, and $b = c$, then $a = c$.
Substitution Property	If $a = b$, then a can be substituted for b in any <u>equation</u> or expression.
Distributive Property	$a(b + c) = ab + ac$.

Algebraic Proof: a proof made up of a series of algebraic statements, where you would use the properties of algebra to justify each step when you solve an equation.

Examples: Using the properties of equality (POE), write a justification for each missing statement/reason.

Statements	Reasons
1. $4(x + 3) = 52$	1. Given
2. $4x + 12 = 52$	2. <u>Distributive POE</u>
3. $4x = 40$	3. <u>Subtraction POE</u>
4. $x = 10$	4. <u>Division POE</u>

Statements	Reasons
1. $2x - 3 = 4x + 12$	1. <u>Given</u>
2. $-2x - 3 = 12$	2. <u>Subtraction POE</u>
3. $-2x = 15$	3. <u>Addition POE</u>
4. $x = -7.5$	4. <u>Division POE</u>

Real-World Example in Science: Write a two-column proof to verify the conjecture below.

The Rankine temperature formula $R = \frac{9}{5}C + 491.69$ converts to the Celsius temperature formula $C = \frac{5}{9}(R - 491.69)$

Statements	Reasons
1. $C = \frac{5}{9}(R - 491.69)$	1. Given
2. $\frac{9}{5}C = R - 491.69$	2. <u>Multiplication POE</u>
3. $\frac{9}{5}C + 491.69 = R$	3. <u>Addition POE</u>
4. $R = \frac{9}{5}C + 491.69$	4. <u>Symmetric POE</u>

2-6 Practice Algebraic Proofs

Name the property of equality that justifies each statement.

1. If $2x - 1 = 15$, then $15 = 2x - 1$. Symmetric POE
2. If $x + 3 = 17$, then $x = 14$. Subtraction POE
3. $x - 2 = x - 2$ Reflexive POE
4. If $7x = 42$, then $x = 6$. Division POE
5. If $x - 5 = 22$, then $x = 27$ Addition POE
6. $2(x + 4) = 2x + 8$ Distributive POE
7. If $x + y = 90$ and $y = 30$, $x + 30 = 90$. Substitution POE
8. If $x = y + 3$ and $y + 3 = 10$, then $x = 10$. Transitive POE

Complete each proof by naming the property that justifies each statement.

9. Given: $3(x - 2) = 9$ Prove: $x = 5$	
Statements:	Reasons:
a. $3(x - 2) = 9$	a. Given
b. $3x - 6 = 9$	b. Distributive POE
c. $3x = 15$	c. Addition POE
d. $x = 5$	d. Division POE

10. Given: $3(x - 4) = \frac{1}{2}x + 8$. Prove: $x = 8$	
Statements:	Reasons:
a. $3(x - 4) = \frac{1}{2}x + 8$	a. Given
b. $3x - 12 = \frac{1}{2}x + 8$	b. Distributive POE
c. $\frac{5}{2}x - 12 = 8$	c. Subtraction POE
d. $\frac{5}{2}x = 20$	d. Addition POE
e. $x = 8$	e. Multiplication POE