Factoring Patterns

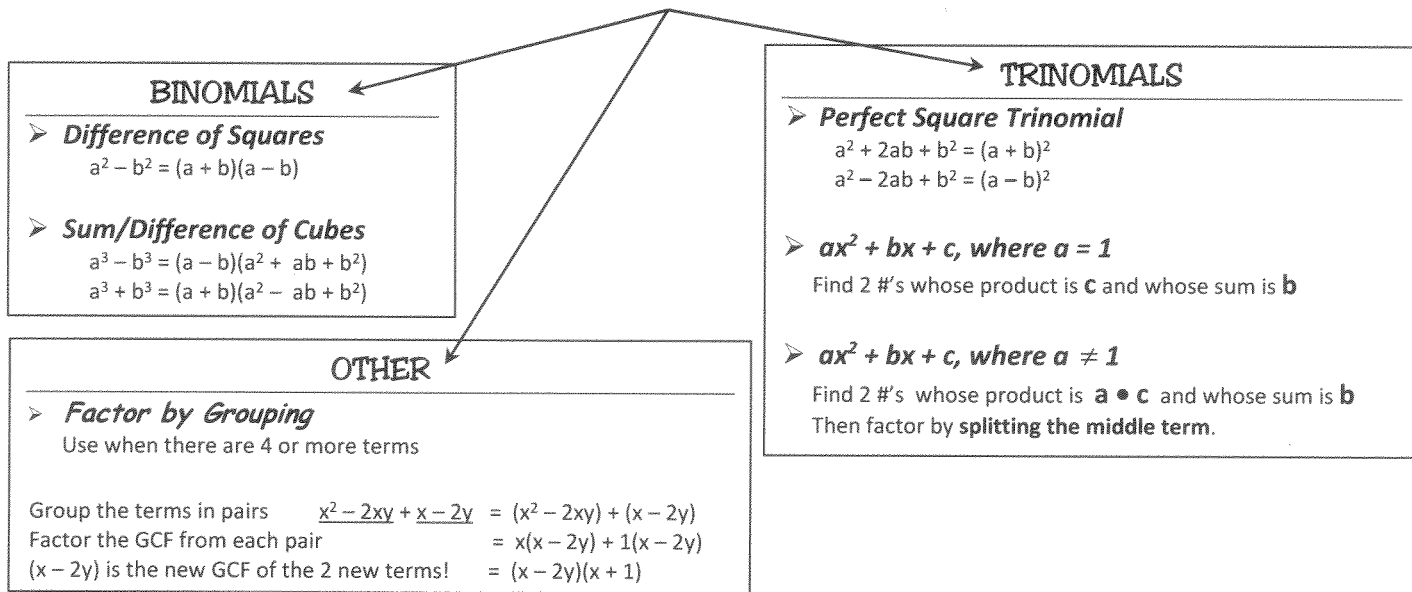To FACTOR a Polynomial
↓
FIRST: FACTOR the GCF
↓
THEN: Look for a PATTERN !

BINOMIALS

➤ **Difference of Squares**
   $a^2 - b^2 = (a + b)(a - b)$

➤ **Sum/Difference of Cubes**
   $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
   $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

TRINOMIALS

➤ **Perfect Square Trinomial**
   $a^2 + 2ab + b^2 = (a + b)^2$
   $a^2 - 2ab + b^2 = (a - b)^2$

➤ **$ax^2 + bx + c$, where $a = 1$**
   Find 2 #'s whose product is **c** and whose sum is **b**

➤ **$ax^2 + bx + c$, where $a \neq 1$**
   Find 2 #'s whose product is **a • c** and whose sum is **b**
   Then factor by **splitting the middle term**.

OTHER

➤ *Factor by Grouping*
   Use when there are 4 or more terms

   Group the terms in pairs $\underline{x^2 - 2xy} + \underline{x - 2y}$ $= (x^2 - 2xy) + (x - 2y)$
   Factor the GCF from each pair $= x(x - 2y) + 1(x - 2y)$
   $(x - 2y)$ is the new GCF of the 2 new terms! $= (x - 2y)(x + 1)$

1-12: Identify the appropriate factoring pattern, then factor each expression completely.

1. $x^2 + 9x + 20$

$(x+4)(x+5)$

2. $2x^2 - 20x + 42$

$2(x^2 - 10x + 21)$
$2(x-7)(x-3)$

3. $12an^2 + 40a^4$

$4a(3n^2 + 10a^3)$

4. $x^2 - 25$

$(x+5)(x-5)$

5. $x^2 + 14x + 49$

$(x+7)^2$

6. $4x^2 - 36$

$4(x^2 - 9)$
$4(x+3)(x-3)$

7. $c^2 + 8c$

$c(c+8)$

8. $-15x^2 - 5x$

$-5x(3x+1)$

9. $3t^2 - 18t + 24$
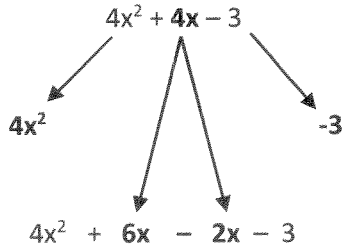
$3(t^2 - 6t + 8)$
$3(t-4)(t-2)$

10. $4x^2 - 9$

$(2x+3)(2x-3)$

11. $x^2 + 12x + 35$

$(x+7)(x+5)$

12. $x^2 + 12x + 36$

$(x+6)^2$

FACTORING by "Splitting the Middle Term"

This method of factoring can be used to factor a trinomial of the form **ax² + bx + c**

$4x^2 + 4x - 3$

$4x^2$ -3

$4x^2 + 6x - 2x - 3$

$\underline{4x^2 + 6x} + \underline{-2x - 3}$

$\underline{\mathbf{2x}(2x+3)} - \underline{\mathbf{1}(2x+3)}$
"term A"   "term B"

$(2x+3)(\mathbf{2x-1})$
$\boxed{4x^2 + 4x - 3 = (2x+3)(2x-1)}$

♦ Always start by factoring the greatest common factor (GCF) from all terms. In this example, there is no GCF for all three terms other than 1.

♦ Multiply the leading coefficient by the constant. Since 4(-3) = -12, find two factors of –12 whose sum is 4.

♦ Since 6 and –2 are the factors that work, split the middle term (4x) into two terms (6x and –2x).

♦ Group the first pair of terms and group the second pair of terms.

♦ Factor the greatest common factor from each group.
♦ We can now look at this expression as an expression with two terms separated by a minus sign. These two terms have a common factor of (2x+ 3).

♦ Last step!: Factor the common factor (2x + 3), and determine that remaining factor is (**2x − 1**).

Another Example: Factor 3x² – 13x – 10

•(-30)	+(-13)
-6 • 5	-1
-10 • 3	-7
-15 • 2	-13 ✔

$= 3x^2 - 15x + 2x - 10$
$= 3x(x-5) + 2(x-5)$
$= (x-5)(3x+2)$

13-17: Try factoring the following trinomials using the "splitting the middle term" method:

13. 2x² – x – 3 $\frac{-6}{-3\cdot 2}\Big|\frac{-1}{}$

$2x^2 - 3x + 2x - 3$
$x(2x-3) + 1(2x-3)$
$(2x-3)(x+1)$

14. 6x² – x – 1 $\frac{-6}{-3\cdot 2}\Big|\frac{-1}{}$

$6x^2 - 3x + 2x - 1$
$3x(2x-1) + 1(2x-1)$
$(2x-1)(3x+1)$

15. 6x² – 39x – 21 $\frac{-14}{14\cdot 1}\Big|\frac{-13}{}$

$3(2x^2 - 13x - 7)$
$3(2x^2 - 14x + x - 7)$
$3(2x(x-7) + 1(x-7))$
$3(x-7)(2x+1)$

16. 2x² + 13x + 6 $\frac{12}{12\cdot 1}\Big|\frac{13}{}$

$2x^2 + 12x + x + 6$
$2x(x+6) + 1(x+6)$
$(x+6)(2x+1)$

17. 4x² + 3x + 1 $\frac{4}{\substack{2\cdot 2 \\ 4\cdot 1}}\Big|\frac{3}{}$

PRIME

18. 16x² – 24x + 9 $\frac{144}{-12\cdot -12}\Big|\frac{-24}{}$

$16x^2 - 12x - 12x + 9$
$4x(4x-3) - 3(4x-3)$
$(4x-3)(4x-3)$
$(4x-3)^2$