

9-6 Solving Rational Equations

Name Master
Date _____ Block _____

Solving a Rational Equation using the LCD

- Multiply each term on both sides of the equation by the LCD (least common denominator) of the terms.
- Simplify and solve the resulting polynomial equation.

EXAMPLE A: Solve: $\frac{3}{x} - \frac{1}{2} = \frac{12}{x}$ The LCD of the denominator is $2x$. So, multiply both sides of the equation by $2x$.

$$2x\left(\frac{3}{x}\right) - 2x\left(\frac{1}{2}\right) = 2x\left(\frac{12}{x}\right)$$

$$6 - x = 24 \longrightarrow \boxed{x = -18}$$

Remember to check for extraneous solutions.

EXAMPLE B: Solve: $\frac{2x+1}{x-4} = \frac{16}{x^2-16} + 3$ The LCD of the denominator is $(x+4)(x-4)$.

Multiply both sides of the equation by this expression.

$$(x+4)(x-4)\left(\frac{2x+1}{x-4}\right) = (x+4)(x-4)\left(\frac{16}{x^2-16}\right) + 3(x+4)(x-4)$$

Reduce where possible.

$$(x+4)(2x-1) = 16 + 3(x^2-16)$$

$$2x^2 + 9x + 4 = 16 + 3x^2 - 48$$

$$0 = x^2 - 9x - 36$$

$$0 = (x-12)(x+3) \longrightarrow \boxed{x = 12, x = -3}$$

Check for extraneous solutions.

Easy way:
Get like fractions on one side & cross multiply!

Easy way:
 $1 = \frac{4-x}{x-4}$
 $1 = -1$
 \emptyset

Solve each equation using the LCD. Check for extraneous solutions.

1. $\frac{3}{x} - \frac{2}{x+1} = \frac{4}{x}$

$$x(x+1)\left(\frac{3}{x}\right) - x(x+1)\left(\frac{2}{x+1}\right) = x(x+1)\left(\frac{4}{x}\right)$$

$$3x+3 - 2x = 4x+4$$

$$x+3 = 4x+4$$

$$-1 = 3x$$

$$\boxed{x = -\frac{1}{3}}$$

Easy way:

$$\frac{-2}{x+1} = \frac{1}{x}$$

$$-2x = x+1$$

$$-3x = 1$$

$$\boxed{x = -\frac{1}{3}}$$

2. $\frac{x}{x-4} + 1 = \frac{4}{x-4}$

$$(x-4)\left(\frac{x}{x-4}\right) + (x-4)(1) = (x-4)\left(\frac{4}{x-4}\right)$$

$$x + x - 4 = 4$$

$$2x = 8$$

$$\boxed{x = 4}$$

FA: \emptyset $x \neq 4!$

3. $\frac{4}{x} - \frac{1}{x+2} = \frac{2}{x}$

$$x(x+2)\left(\frac{4}{x}\right) - x(x+2)\left(\frac{1}{x+2}\right) = x(x+2)\left(\frac{2}{x}\right)$$

$$4x+8 - x = 2x+4$$

$$3x+8 = 2x+4$$

$$\boxed{x = -4}$$

Easy way:

$$\frac{-1}{x+2} = \frac{-2}{x}$$

$$-x = -2(x+2)$$

$$-x = -2x - 4$$

$$\boxed{x = -4}$$

4. $\frac{1}{x+2} + \frac{1}{x+2} = \frac{4}{x^2-4}$

$$(x+2)(x-2)\left(\frac{1}{x+2}\right) + (x+2)(x-2)\left(\frac{1}{x+2}\right) = (x+2)(x-2)\left(\frac{4}{x^2-4}\right)$$

$$x-2 + x-2 = 4$$

$$2x-4 = 4$$

$$2x = 8$$

$$\boxed{x = 4}$$

Easy way:

$$\frac{2}{x+2} = \frac{4}{x^2-4}$$

$$2(x^2-4) = 4(x+2)$$

$$2x^2 - 8 = 4x + 8$$

$$2x^2 - 4x - 16 = 0$$

$$2x^2 - 4x - 16 = 0$$

2 is the answer
FA: $x=4$

Solving a Rational Equation by CROSS-MULTIPLYING

Use cross-multiplying when each side of the equation is a single rational expression.

EXAMPLE Solve:

$$\frac{3}{x^2 + 4x} = \frac{1}{x + 4}$$

$$3(x + 4) = 1(x^2 + 4x)$$

$$3x + 12 = x^2 + 4x$$

$$0 = x^2 + x - 12$$

$$0 = (x + 4)(x - 3)$$

$$x = -4, x = 3$$

The solution $x = -4$ is an extraneous solution since if $x = -4$ in the original equation, the denominator would be zero.

So the only solution is

$$x = 3$$

Solve each equation by cross-multiplying. Check for extraneous solutions.

5. $\frac{7}{x+3} = \frac{x}{4}$

$$x(x+3) = 28$$

$$x^2 + 3x = 28$$

$$x^2 + 3x - 28 = 0$$

$$(x+7)(x-4) = 0$$

$$x = -7, 4$$

7. $\frac{2x-3}{x+3} = \frac{3x}{x+4}$

$$(2x-3)(x+4) = 3x(x+3)$$

$$2x^2 + 5x - 12 = 3x^2 + 9x$$

$$0 = x^2 + 4x + 12$$

$$\frac{-4 \pm \sqrt{6-4(12)}}{2} = \frac{-4 \pm \sqrt{-32}}{2}$$

$$-2 \pm 2i\sqrt{2}$$

Easy way

if $\frac{A}{B} = \frac{C}{B}$
then $A = C!$

6. $\frac{x}{x-3} = \frac{6}{x-3}$

$$x = 6$$

$$x^2 - 3x = 6x - 18$$

$$x^2 - 9x + 18 = 0$$

$$(x-6)(x-3) = 0$$

$$x = 6, x = 3$$

but $x \neq 3$, so $x = 6$

8. $\frac{x}{x^2-10} = \frac{3}{2x+1}$

$$2x^2 + x = 3x^2 - 30$$

$$0 = x^2 - x - 30$$

$$0 = (x-6)(x+5) = 0$$

$$x = 6, -5$$

Solve each equation using the most appropriate method. Check for extraneous solutions. or Easy way

9. $\frac{3}{x-1} - 6 = \frac{5x}{x-1}$ ($-\frac{3}{x-1}$ from both sides)

$$-6 = \frac{5x-3}{x-1}$$

$$-6x + 6 = 5x - 3$$

$$-11x = -9$$

$$x = \frac{9}{11}$$

10. $\frac{5x-7}{x-2} = \frac{8}{x-2}$

cross multiplies

$$(5x-7)(x-2) = 8(x-2)$$

$$5x^2 - 17x + 14 = 8x - 16$$

$$5x^2 - 25x + 30 = 0$$

$$5(x^2 - 5x + 6) = 0$$

$$5(x-2)(x-3) = 0$$

$x = 2, 3$, but $x \neq 2$, so $x = 3$

$$5x - 7 = 8$$

$$5x = 15$$

$$x = 3$$

Add these $(x-2)(x+3)$

$$11. \frac{1}{x-2} + \frac{1}{x+3} = \frac{5}{x^2+x-6}$$

$$\frac{x+3+x-2}{(x-2)(x+3)} = \frac{5}{(x-2)(x+3)}$$

$$2x+1=5$$

$$2x=4$$

$$\cancel{x=2}$$

extr.

$x \neq 2$ asymptote

$$FA = \emptyset$$

Multiply by $4x$ (LCD)

$$12. \left(\frac{12}{x} + \frac{3}{4} = \frac{3}{2} \right)$$

$$48 + 3x = 6x$$

$$48 = 3x$$

$$16 = x$$

$$13. \frac{x^2}{8} - 4 = \frac{x}{2} \quad -4 = \frac{-32}{8}$$

$$\frac{x^2 - 32}{8} = \frac{x}{2}$$

$$2x^2 - 64 = 8x$$

$$2x^2 - 8x - 64 = 0$$

$$2(x^2 - 4x - 32) = 0$$

$$2(x-8)(x+4) = 0$$

$$x = 8, -4$$

$$14. \frac{x+10}{x^2-2} = \frac{4}{x}$$

cross multiply: $x^2 + 10x = 4x^2 - 8$

$$0 = 3x^2 - 10x - 8$$

$$(3x+2)(x-4)$$

$$x = -\frac{2}{3}, 4$$

Subtract

$$15. \frac{x}{x+2} + x = \frac{5x+8}{x+2}$$

$$x = \frac{4x+8}{x+2}$$

$$x^2 + 2x = 4x + 8$$

$$x^2 - 2x - 8 = 0$$

$$(x-4)(x+2) = 0$$

$$x = 4, -2$$

$$x = 4$$

Subtract over!

$$16. \frac{5}{x-5} = \frac{x}{x-5} - 1$$

$$\frac{5-x}{x-5} = -1$$

$$-1 = -1$$

$$\mathbb{R}, \text{ but } x \neq 5$$

$$\text{so } (-\infty, 5) \cup (5, \infty)$$

$$17. \frac{1}{3x-2} + \frac{5}{x} = 0$$

$$\frac{1}{3x-2} = -\frac{5}{x}$$

$$x = -15x + 10$$

$$16x = 10$$

$$x = \frac{10}{16}$$

$$\boxed{x = \frac{5}{8}} = 0.625 \quad \text{OR}$$

$$18. \frac{1}{x+3} = \frac{2}{x} - \frac{3}{4x}$$

Multiply by
LCD:
 $4x(x+3)$

$$4x = 2 \cdot 4(x+3) - 3(x+3)$$

$$4x = 8x + 24 - 3x - 9$$

$$4x = 5x + 15$$

$$-x = 15$$

$$\boxed{x = -15}$$

$$19. \frac{5}{x+6} = \frac{9x+6}{x^2+6x} + \frac{2}{x}$$

Mult by LCD
 $x(x+6)$

$$5x = 9x + 6 + 2(x+6)$$

$$5x = 9x + 6 + 2x + 12$$

$$5x = 11x + 18$$

$$-6x = 18$$

$$\boxed{x = -3}$$

$$20. \frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$$

Mult. by LCD:
 $(x-1)(x-2)(x+1)$

$$6(x-2)(x+1) = 4(x-1)(x+1) + 2(x-1)(x-2)$$

$$6(x^2 - x - 2) = 4(x^2 - 1) + 2(x^2 - 3x + 2)$$

$$6x^2 - 6x - 12 = 4x^2 - 4 + 2x^2 - 6x + 4$$

$$-12 = 0$$



$$21. \frac{x+1}{x-3} = 4 - \frac{12}{x^2-2x-3}$$

Mult by LCD
 $(x+1)(x-3)$

$$(x+1)(x+1) = 4(x+1)(x-3) - 12$$

$$x^2 + 2x + 1 = 4(x^2 - 2x - 3) - 12$$

$$x^2 + 2x + 1 = 4x^2 - 8x - 12 - 12$$

$$x^2 + 2x + 1 = 4x^2 - 8x - 24$$

$$0 = 3x^2 - 10x - 25$$

$$(3x+5)(x-5)$$

$$\boxed{x = -\frac{5}{3}, 5}$$

$$22. \frac{1}{x-1} = \frac{2}{x+1} - \frac{1}{x+3}$$

Mult. by LCD:
 $(x-1)(x+1)(x+3)$

$$(x+1)(x+3) = 2(x-1)(x+3) - 1(x-1)(x+1)$$

$$x^2 + 4x + 3 = 2(x^2 + 2x - 3) - (x^2 - 1)$$

$$x^2 + 4x + 3 = 2x^2 + 4x - 6 - x^2 + 1$$

$$x^2 + 4x + 3 = x^2 + 4x - 5$$

$$3 = -5$$

