Day 06 Solving Rational Equations

Solving a Rational Equation using the LCD

1. Multiply each term on both sides of the equation by the LCD (least common denominator) of the terms.
2. Simplify and solve the resulting polynomial equation.

EXAMPLE A: Solve: $\dfrac{3}{x} - \dfrac{1}{2} = \dfrac{12}{x}$ **The LCD of the denominator is 2x. So, multiply both sides of the equation by 2x.**

$$2x\left(\dfrac{3}{x}\right) - 2x\left(\dfrac{1}{2}\right) = 2x\left(\dfrac{12}{x}\right)$$

$$6 - x = 24 \longrightarrow \boxed{x = -18}$$ **Remember to check for extraneous solutions.**

EXAMPLE B: Solve: $\dfrac{2x+1}{x-4} = \dfrac{16}{x^2-16} + 3$ **The LCD of the denominator is (x + 4)(x - 4).**

Multiply both sides of the equation by this expression.

$$(x+4)(x-4)\left(\dfrac{2x+1}{x-4}\right) = (x+4)(x-4)\left(\dfrac{16}{x^2-16}\right) + 3(x+4)(x-4) \text{ Reduce where possible.}$$

$$(x + 4)(2x - 1) = 16 + 3(x^2 - 16)$$
$$2x^2 + 9x + 4 = 16 + 3x^2 - 48$$
$$0 = x^2 - 9x - 36$$
$$0 = (x - 12)(x + 3) \longrightarrow \boxed{x = 12, \ x = -3} \quad \text{Check for extraneous solutions.}$$

Solve each equation using the LCD. Check for extraneous solutions.

1. $\dfrac{3}{x} - \dfrac{2}{x+1} = \dfrac{4}{x}$

2. $\dfrac{x}{x-4} + 1 = \dfrac{4}{x-4}$

3. $\dfrac{4}{x} - \dfrac{1}{x+2} = \dfrac{2}{x}$

4. $\dfrac{1}{x+2} + \dfrac{1}{x+2} = \dfrac{4}{x^2-4}$

Solving a Rational Equation by CROSS-MULTIPLYING

Use cross-multiplying when each side of the equation is a single rational expression.

EXAMPLE Solve: $\dfrac{3}{x^2 + 4x} = \dfrac{1}{x + 4}$

$$3(x + 4) = 1(x^2 + 4x)$$
$$3x + 12 = x^2 + 4x$$
$$0 = x^2 + x - 12$$
$$0 = (x + 4)(x - 3)$$
$$x = -4 \ , \ x = 3$$

The solution $x = -4$ is an extraneous solution since if $x = -4$ in the original equation, the denominator would be zero.

So the only solution is $\boxed{x = 3}$

Solve each equation by cross-multiplying. Check for extraneous solutions.

5. $\dfrac{7}{x + 3} = \dfrac{x}{4}$

6. $\dfrac{x}{x - 3} = \dfrac{6}{x - 3}$

7. $\dfrac{2x - 3}{x + 3} = \dfrac{3x}{x + 4}$

8. $\dfrac{x}{x^2 - 10} = \dfrac{3}{2x + 1}$

Solve each equation using the most appropriate method. Check for extraneous solutions.

9. $\dfrac{3}{x - 1} - 6 = \dfrac{5x}{x - 1}$

10. $\dfrac{5x - 7}{x - 2} = \dfrac{8}{x - 2}$

11. $\dfrac{1}{x-2} + \dfrac{1}{x+3} = \dfrac{5}{x^2+x-6}$

12. $\dfrac{12}{x} + \dfrac{3}{4} = \dfrac{3}{2}$

13. $\dfrac{x^2}{8} - 4 = \dfrac{x}{2}$

14. $\dfrac{x+10}{x^2-2} = \dfrac{4}{x}$

15. $\dfrac{x}{x+2} + x = \dfrac{5x+8}{x+2}$

16. $\dfrac{5}{x-5} = \dfrac{x}{x-5} - 1$

17. $\dfrac{1}{3x-2} + \dfrac{5}{x} = 0$

18. $\dfrac{1}{x+3} = \dfrac{2}{x} - \dfrac{3}{4x}$

19. $\dfrac{5}{x+6} = \dfrac{9x+6}{x^2+6x} + \dfrac{2}{x}$

20. $\dfrac{6}{x-1} = \dfrac{4}{x-2} + \dfrac{2}{x+1}$

21. $\dfrac{x+1}{x-3} = 4 - \dfrac{12}{x^2-2x-3}$

22. $\dfrac{1}{x-1} = \dfrac{2}{x+1} - \dfrac{1}{x+3}$