

9-6 Solving Rational Equations

Name Master
Date _____ Block _____

Solving a Rational Equation using the LCD

- Multiply each term on both sides of the equation by the LCD (least common denominator) of the terms.
- Simplify and solve the resulting polynomial equation.

EXAMPLE A: Solve: $\frac{3}{x} - \frac{1}{2} = \frac{12}{x}$ The LCD of the denominator is $2x$. So, multiply both sides of the equation by $2x$.

$$2x\left(\frac{3}{x}\right) - 2x\left(\frac{1}{2}\right) = 2x\left(\frac{12}{x}\right)$$

$$6 - x = 24 \longrightarrow$$

$$x = -18$$

Remember to check for extraneous solutions.

EXAMPLE B: Solve: $\frac{2x+1}{x-4} = \frac{16}{x^2-16} + 3$ The LCD of the denominator is $(x+4)(x-4)$.

Multiply both sides of the equation by this expression.

$$(x+4)(x-4)\left(\frac{2x+1}{x-4}\right) = (x+4)(x-4)\left(\frac{16}{x^2-16}\right) + 3(x+4)(x-4) \text{ Reduce where possible.}$$

$$(x+4)(2x-1) = 16 + 3(x^2-16)$$

$$2x^2 + 9x + 4 = 16 + 3x^2 - 48$$

$$0 = x^2 - 9x - 36$$

$$0 = (x-12)(x+3) \longrightarrow$$

$$x = 12, x = -3$$

Check for extraneous solutions.

*Easy way:
Get like fractions
on one side &
cross multiply!*

*Easy way:
 $1 = \frac{4-x}{x-4}$
 $1 = -1$
 \emptyset*

Solve each equation using the LCD. Check for extraneous solutions.

1. $\frac{3}{x} - \frac{2}{x+1} = \frac{4}{x}$

$$x(x+1)\left(\frac{3}{x}\right) - x(x+1)\left(\frac{2}{x+1}\right) = x(x+1)\left(\frac{4}{x}\right)$$

$$3x+3-2x = 4x+4$$

$$x+3 = 4x+4$$

$$-1 = 3x$$

$$x = -\frac{1}{3}$$

Easy way:

$$\frac{-2}{x+1} = \frac{1}{x}$$

$$-2x = x+1$$

$$-3x = 1$$

$$x = -\frac{1}{3}$$

2. $\frac{x}{x-4} + 1 = \frac{4}{x-4}$

$$(x-4)\left(\frac{x}{x-4}\right) + (x-4)(1) = (x-4)\left(\frac{4}{x-4}\right)$$

$$x + x - 4 = 4$$

$$2x = 8$$

$$x = 4$$

FA: \emptyset $x \neq 4!$

3. $\frac{4}{x} - \frac{1}{x+2} = \frac{2}{x}$

$$x(x+2)\left(\frac{4}{x}\right) - x(x+2)\left(\frac{1}{x+2}\right) = x(x+2)\left(\frac{2}{x}\right)$$

$$4x+8-x = 2x+4$$

$$3x+8 = 2x+4$$

$$x = -4$$

Easy way:

$$\frac{-1}{x+2} = \frac{-2}{x}$$

$$-x = -2(x+2)$$

$$-x = -2x - 4$$

$$x = -4$$

4. $\frac{1}{x+2} + \frac{1}{x+2} = \frac{4}{x^2-4}$

$$(x+2)(x-2)\left(\frac{1}{x+2}\right) + (x+2)(x-2)\left(\frac{1}{x+2}\right) = (x+2)(x-2)\left(\frac{4}{x^2-4}\right)$$

$$x-2 + x-2 = 4$$

$$2x-4 = 4$$

$$2x = 8$$

$$x = 4$$

Easy way:

$$\frac{2}{x+2} = \frac{4}{x^2-4}$$

$$2(x-2) = 4(x+2)$$

$$2x-8 = 4x+8$$

$$2x^2-8 = 4x^2+8$$

$$2x^2-4x-16 = 0$$

$$2x^2-4x-16 = 0$$

-2 in fact FA $x=4$

Solving a Rational Equation by CROSS-MULTIPLYING

Use cross-multiplying when each side of the equation is a single rational expression.

EXAMPLE Solve: $\frac{3}{x^2 + 4x} = \frac{1}{x + 4}$

$$\begin{aligned} 3(x + 4) &= 1(x^2 + 4x) \\ 3x + 12 &= x^2 + 4x \\ 0 &= x^2 + x - 12 \\ 0 &= (x + 4)(x - 3) \\ x &= -4, x = 3 \end{aligned}$$

The solution $x = -4$ is an extraneous solution since if $x = -4$ in the original equation, the denominator would be zero.

So the only solution is x = 3

Solve each equation by cross-multiplying. Check for extraneous solutions.

5. $\frac{7}{x+3} = \frac{x}{4}$

$$\begin{aligned} x(x+3) &= 28 \\ x^2 + 3x &= 28 \\ x^2 + 3x - 28 &= 0 \\ (x+7)(x-4) &= 0 \\ x &= -7, 4 \end{aligned}$$

Easy way
if $\frac{A}{B} = \frac{C}{B}$
then $A = C!$ → x = 6

$$\begin{aligned} x^2 - 3x &= 6x - 18 \\ x^2 - 9x + 18 &= 0 \\ (x-6)(x-3) &= 0 \\ x &= 6, x = 3 \\ \text{but } x \neq 3, \text{ so } x &= 6 \end{aligned}$$

7. $\frac{2x-3}{x+3} = \frac{3x}{x+4}$

$$\begin{aligned} (2x-3)(x+4) &= 3x(x+3) \\ 2x^2 + 5x - 12 &= 3x^2 + 9x \\ 0 &= x^2 + 4x + 12 \end{aligned}$$

$$\begin{aligned} x^2 + 4x + \frac{4}{4} &= -12 + \frac{4}{4} \\ (x+2)^2 &= -8 \end{aligned}$$

$$\begin{aligned} x+2 &= \pm i\sqrt{8} \\ x &= -2 \pm 2i\sqrt{2} \end{aligned}$$

8. $\frac{x}{x^2-10} = \frac{3}{2x+1}$

$$\begin{aligned} 2x^2 + x &= 3x^2 - 30 \\ 0 &= x^2 - x - 30 \\ 0 &= (x-6)(x+5) = 0 \end{aligned}$$

x = 6, -5

Solve each equation using the most appropriate method. Check for extraneous solutions.

9. $\frac{3}{x-1} - 6 = \frac{5x}{x-1}$

(-3/x-1 from both sides)

$$\begin{aligned} -6 &= \frac{5x-3}{x-1} \\ -6x+6 &= 5x-3 \\ -11x &= -9 \\ x &= \frac{9}{11} \end{aligned}$$

10. $\frac{5x-7}{x-2} = \frac{8}{x-2}$

cross multiply

$$\begin{aligned} (5x-7)(x-2) &= 8(x-2) \\ 5x^2 - 17x + 14 &= 8x - 16 \\ 5x^2 - 25x + 30 &= 0 \\ 5(x^2 - 5x + 6) &= 0 \\ 5(x-2)(x-3) &= 0 \end{aligned}$$

or Easy way
 $5x-7=8$
 $5x=15$
 $x=3$

$x = 2, 3$, but $x \neq 2$, so x = 3

Add these $(x-2)(x+3)$

11. $\frac{1}{x-2} + \frac{1}{x+3} = \frac{5}{x^2+x-6}$

$\frac{x+3+x-2}{(x-2)(x+3)} = \frac{5}{(x-2)(x+3)}$

$2x+1=5$

$2x=4$

~~$x=2$~~

$x \neq 2$ asymptote

extr.

$FA = \emptyset$

Multiply by $4x$ (LCD)

12. $\left(\frac{12}{x} + \frac{3}{4} = \frac{3}{2}\right)$

$48 + 3x = 6x$

$48 = 3x$

$16 = x$

13. $\frac{x^2}{8} - 4 = \frac{x}{2}$ $-4 = \frac{-32}{8}$

$\frac{x^2-32}{8} = \frac{x}{2}$

$2x^2 - 64 = 8x$

$2x^2 - 8x - 64 = 0$

$2(x^2 - 4x - 32) = 0$

$2(x-8)(x+4) = 0$

$x = 8, -4$

14. $\frac{x+10}{x^2-2} = \frac{4}{x}$

Cross multiply: $x^2+10x = 4x^2-8$

$0 = 3x^2 - 10x - 8$

$0 = (3x+2)(x-4)$

$x = -\frac{2}{3}, 4$

$\begin{array}{r} -24 \quad -10 \\ -12 \cdot 2 \end{array}$

$\frac{(x-12)(x+2)}{3}$

$\frac{(x-4)(3x+2)}{3}$

Subtract

15. $\frac{x}{x+2} - x = \frac{5x+8}{x+2}$

$x = \frac{4x+8}{x+2}$

$x^2+2x = 4x+8$

$x^2-2x-8 = 0$

$(x-4)(x+2) = 0$

$x = 4, -2$

$x = 4$

Subtract over!

16. $\frac{5}{x-5} = \frac{x}{x-5} - 1$

$\frac{5-x}{x-5} = -1$

$-1 = -1$

\mathbb{R} , but $x \neq 5$
so $(-\infty, 5) \cup (5, \infty)$

$$17. \frac{1}{3x-2} + \frac{5}{x} = 0$$

$$\frac{1}{3x-2} = -\frac{5}{x}$$

$$x = -15x + 10$$

$$16x = 10$$

$$x = \frac{10}{16}$$

$$x = \frac{5}{8} = 0.625 \quad \text{OR}$$

Multiply by
LCD:
 $4x(x+3)$

$$18. \frac{1}{x+3} = \frac{2}{x} - \frac{3}{4x}$$

$$4x = 2 \cdot 4(x+3) - 3(x+3)$$

$$4x = 8x + 24 - 3x - 9$$

$$4x = 5x + 15$$

$$-x = 15$$

$$x = -15$$

Mult. by LCD:
 $(x-1)(x-2)(x+1)$

$$19. \frac{5}{x+6} = \frac{9x+6}{x^2+6x} + \frac{2}{x}$$

$$x(x+6)$$

$$5x = 9x + 6 + 2(x+6)$$

$$5x = 9x + 6 + 2x + 12$$

$$5x = 11x + 18$$

$$-6x = 18$$

$$x = -3$$

$$20. \frac{6}{x-1} = \frac{4}{x-2} + \frac{2}{x+1}$$

$$6(x-2)(x+1) = 4(x-1)(x+1) + 2(x-1)(x-2)$$

$$6(x^2 - x - 2) = 4(x^2 - 1) + 2(x^2 - 3x + 2)$$

$$6x^2 - 6x - 12 = 4x^2 - 4 + 2x^2 - 6x + 4$$

$$-12 = 0$$



$$21. \frac{x+1}{x-3} = 4 - \frac{12}{x^2-2x-3}$$

$$(x+1)(x-3)$$

$$(x+1)(x+1) = 4(x+1)(x-3) - 12$$

$$x^2 + 2x + 1 = 4(x^2 - 2x - 3) - 12$$

$$x^2 + 2x + 1 = 4x^2 - 8x - 12 - 12$$

$$x^2 + 2x + 1 = 4x^2 - 8x - 24$$

$$0 = 3x^2 - 10x - 25$$

$$(3x+5)(x-5)$$

$$x = -\frac{5}{3}, 5$$

Mult. by LCD:
 $(x-1)(x+1)(x+3)$

$$22. \frac{1}{x-1} = \frac{2}{x+1} - \frac{1}{x+3}$$

$$(x+1)(x+3) = 2(x-1)(x+3) - 1(x-1)(x+1)$$

$$x^2 + 4x + 3 = 2(x^2 + 2x - 3) - (x^2 - 1)$$

$$x^2 + 4x + 3 = 2x^2 + 4x - 6 - x^2 + 1$$

$$x^2 + 4x + 3 = x^2 + 4x - 5$$

$$3 = -5$$

