

Day 07 Writing Equations of Polynomial Functions

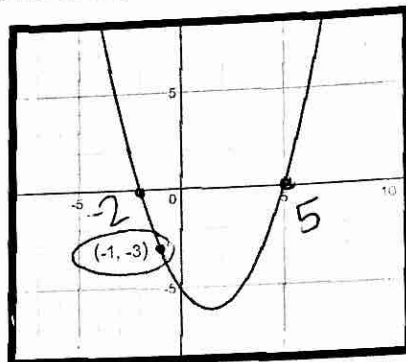
Learning Target 4: I can write the equation of a polynomial function given its zeros/roots or graph.

INTRO: Write the standard form of the given quadratic function.

$$y = a(x+2)(x-5) \rightarrow y = \frac{1}{2}(x+2)(x-5)$$

$$= \frac{1}{2}(x^2 - 3x - 10)$$

$$y = \frac{1}{2}x^2 - \frac{3}{2}x - 5$$



What are the proper steps to write an equation when given the roots?

1. Write the equation in intercept form w/ a as a variable.
2. Plug in a point to find a.
3. Replace a with its value.
4. Simplify the equation

1-4: Write the standard form of the polynomial function with the given roots. Assume that a = 1.

1. $x = \{-4, 0, 4, 2\}$

$$f(x) = (x+4)(x)(x-4)(x-2)$$

$$= x(x^2 - 16)(x-2)$$

$$= (x^3 - 16x)(x-2)$$

$$f(x) = x^4 - 2x^3 - 16x^2 + 32x$$

2. $x = \{3i, -\frac{3}{2}\}$

$$f(x) = (x^2 + 9)(2x + 3)$$

$$f(x) = 2x^3 + 3x^2 + 18x + 27$$

3. $x = \{8, i, -i\}$

$$f(x) = (x-8)(x^2 + 1)$$

$$= x^3 + x - 8x^2 - 8$$

$$f(x) = x^3 - 8x^2 + x - 8$$

4. $x = \{3, 2, 1+i\}$

$$f(x) = (x-3)(x-2)(x-1-i)(x-1+i)$$

$$= (x-3)(x-2)(x^2 - 2x + 2)$$

$$= (x^2 - 5x + 6)(x^2 - 2x + 2)$$

$$= x^4 - 2x^3 + 2x^2 - 5x^3 + 10x^2 - 10x + 6x^2 - 12x + 12$$

$$f(x) = x^4 - 7x^3 + 18x^2 - 22x + 12$$

$$x = 1 \pm i$$

$$x - 1 = \pm i$$

$$(x-1)^2 = (\pm i)^2$$

$$(x-1)^2 = -1$$

$$(x-1)^2 + 1$$

$$x^2 - 2x + 1 + 1$$

$$x^2 - 2x + 2$$

5-7: Write the factored form of the polynomial function described. You must find a !

5. The function that contains the point $(-1, 160)$ and has roots $1, -\frac{2}{3}, 2i$, and a double root at -3 .

$$f(x) = a(x-1)(3x+2)(x^2+4)(x+3)^2$$

$$160 = a(-1-1)(-3+2)(1+4)(-1+3)^2$$

$$160 = a(-2)(-1)(5)(4)$$

$$160 = 40a$$

$$4 = a$$

$$f(x) = 4(x-1)(3x+2)(x^2+4)(x+3)^2$$

6. The function has roots $4, i$, and $\frac{5}{2}$, and also has a y -intercept of -60 .

$$f(x) = a(x-4)(x^2+1)(2x-5)$$

$$-60 = a(-4)(1)(-5)$$

$$-60 = 20a$$

$$-3 = a$$

$$f(x) = -3(x-4)(x^2+1)(2x-5)$$

7. The function has a tangent at 5 , roots at $\pm 3i$, and contains the point $(4, -50)$.

$$f(x) = a(x-5)^2(x^2+9)$$

$$-50 = a(4-5)^2(16+9)$$

$$-50 = a(-1)^2(25)$$

$$-50 = 25a$$

$$-2 = a$$

$$f(x) = -2(x-5)^2(x^2+9)$$

8-9: REVIEW: Find all of the zeros of each function.

8. $f(x) = x^4 - 3x^3 + 5x^2 - 27x - 36$ $-1, 4$

$$\begin{array}{r|rrrrr} -1 & 1 & -3 & 5 & -27 & -36 \\ & \downarrow & -1 & 4 & -9 & 36 \\ \hline & 1 & -4 & 9 & -36 & 0 \end{array} \quad (x+1)(x^3-4x^2+9x-36)$$

$$\begin{array}{r|rrrr} 4 & 1 & -4 & 9 & -36 & 0 \\ & \downarrow & 4 & 0 & 36 & \\ \hline & 1 & 0 & 9 & 0 & \end{array} \quad (x+1)(x-4)(x^2+9) = 0$$

$$(x+1)(x-4)(x^2+9) = 0$$

$$\downarrow$$

$$x^2 = -9$$

$$x = \pm\sqrt{-9}$$

$$\boxed{-1, 4, \pm 3i}$$

9. $g(x) = x^3 - 11x^2 + 46x - 80$ 5

$$\begin{array}{r|rrrr} 5 & 1 & -11 & 46 & -80 \\ & \downarrow & 5 & -30 & 80 \\ \hline & 1 & -6 & 16 & 0 \end{array}$$

$$(x-5)(x^2-6x+16)$$

$$x^2-6x+9 = -16+9$$

$$(x-3)^2 = -7$$

$$x-3 = \pm i\sqrt{7}$$

$$x = 3 \pm i\sqrt{7}$$

$$\boxed{5, 3 \pm i\sqrt{7}}$$