

4  
SHOW ALL WORK AND COME PREPARED TO ACE THE TEST! *May the FORCE be with you!*

State whether  $x$  and  $y$  show *direct variation*, *inverse variation* or *neither*.

1.  $xy = 20$

$$y = \frac{20}{x}$$

1. Inverse

2.  $y - 3 = x$

$$y = x + 3$$

2. Neither

3.  $\frac{y}{6} = x$

$$y = 6x$$

3. Direct

4.

x	y
-4	-1.6
-2	-0.8
1	0.4
2	0.8

$$\frac{y}{x} = .4$$

$$y = .4x$$

4. Direct

For each variation problem, do the following: a) Find the constant of variation  $k$ , b) Write an equation that relates the variables, and c) Answer the question. Show all work used to derive your answer.

5. If  $y$  varies directly as  $z$  and inversely as  $x$ , and  $y = -18$  and  $z = 3$  when  $x = 6$ , find  $y$  when  $x = 5$  and  $z = -5$ .

$$y = \frac{kz}{x} \quad -18 = \frac{k \cdot 3}{6}$$

$$y = \frac{-36z}{x}$$

a.  $k = -36$

b.  $y = \frac{-36z}{x}$

c.  $y = 36$

6.  $M$  varies jointly as the values of  $p$  and  $q$ . If  $M = 88$  when  $p = 4$  and  $q = .4$ , find  $M$  when  $p = 8$  and  $q = 1.2$ .

$$M = kpg \quad 88 = k(4)(.4)$$

$$88 = 1.6k$$

$$M = 55(8)(1.2)$$

a.  $k = 55$

b.  $M = 55pg$

c.  $M = 528$

7. A company has found that the monthly demand  $d$  for one of its products varies inversely with the price  $p$  of the product. When the price is \$12.50, the demand is 12,000 units. What will the demand be if the price is reduced to \$12.00?

$$d = \frac{k}{p} \quad 12000 = \frac{k}{12.50}$$

$$d = \frac{150,000}{12}$$

a.  $k = 150,000$

b.  $d = \frac{150,000}{p}$

c.  $d = 12,500$  units

8. In a factory, the profit,  $P$ , varies directly with the inventory,  $I$ . If the Profit was \$100 when the inventory was \$20, find the Profit when the inventory is \$50.

$$P = kI \quad 100 = k20$$

$$P = 5(50)$$

a.  $k = 5$

b.  $P = 5I$

c.  $P = \$250.00$

9. The volume,  $V$ , of a gas varies inversely as the pressure,  $p$ , in a container. If the volume of a gas is 200cc when the pressure is 1.6 liters per square centimeter, find the volume (to the nearest tenth) when the pressure is 2.8 liters per sq centimeter.

$$V = \frac{k}{p} \quad 200 = \frac{k}{1.6}$$

$$V = \frac{320}{2.8}$$

a.  $k = 320$

b.  $V = \frac{320}{p}$

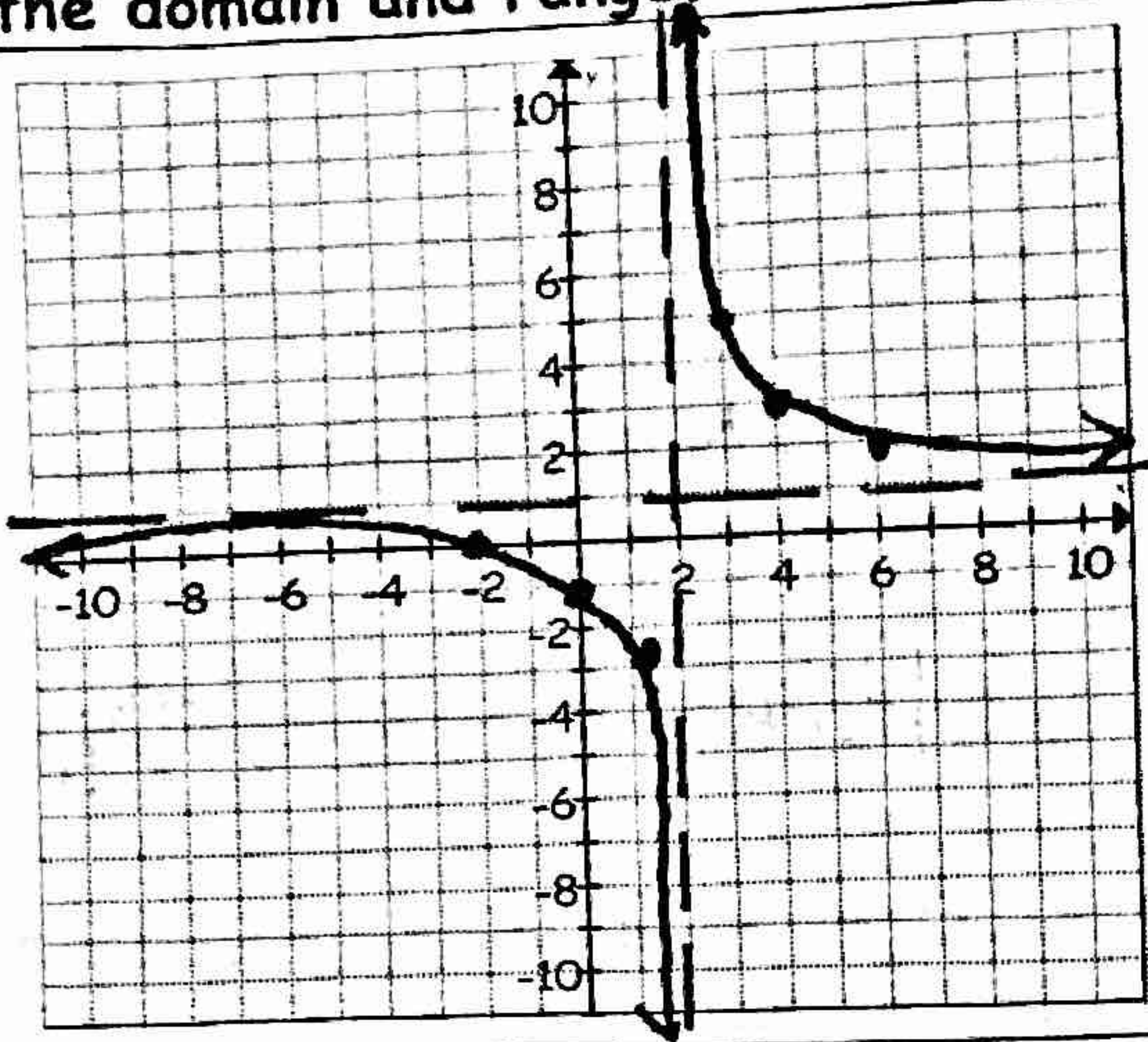
c.  $V = 114.3$  cc



Graph each of the following functions and provide the indicated information below. Plot at least two points on each branch of the graph. Use appropriate notation for equations and points, and use interval notation for the domain and range.

10.  
 $y = \frac{4}{x-2} + 1$

$0 = \frac{4}{x-2} + 1$   
 $-1 = \frac{4}{x-2}$   
 $-x+2=4$   
 $-x=2$   
 $x=-2$



V asymptote(s):  
 $x=2$

H asymptote:  
 $y=1$

SLANT asymptote:  
 none

Hole(s):  
 none

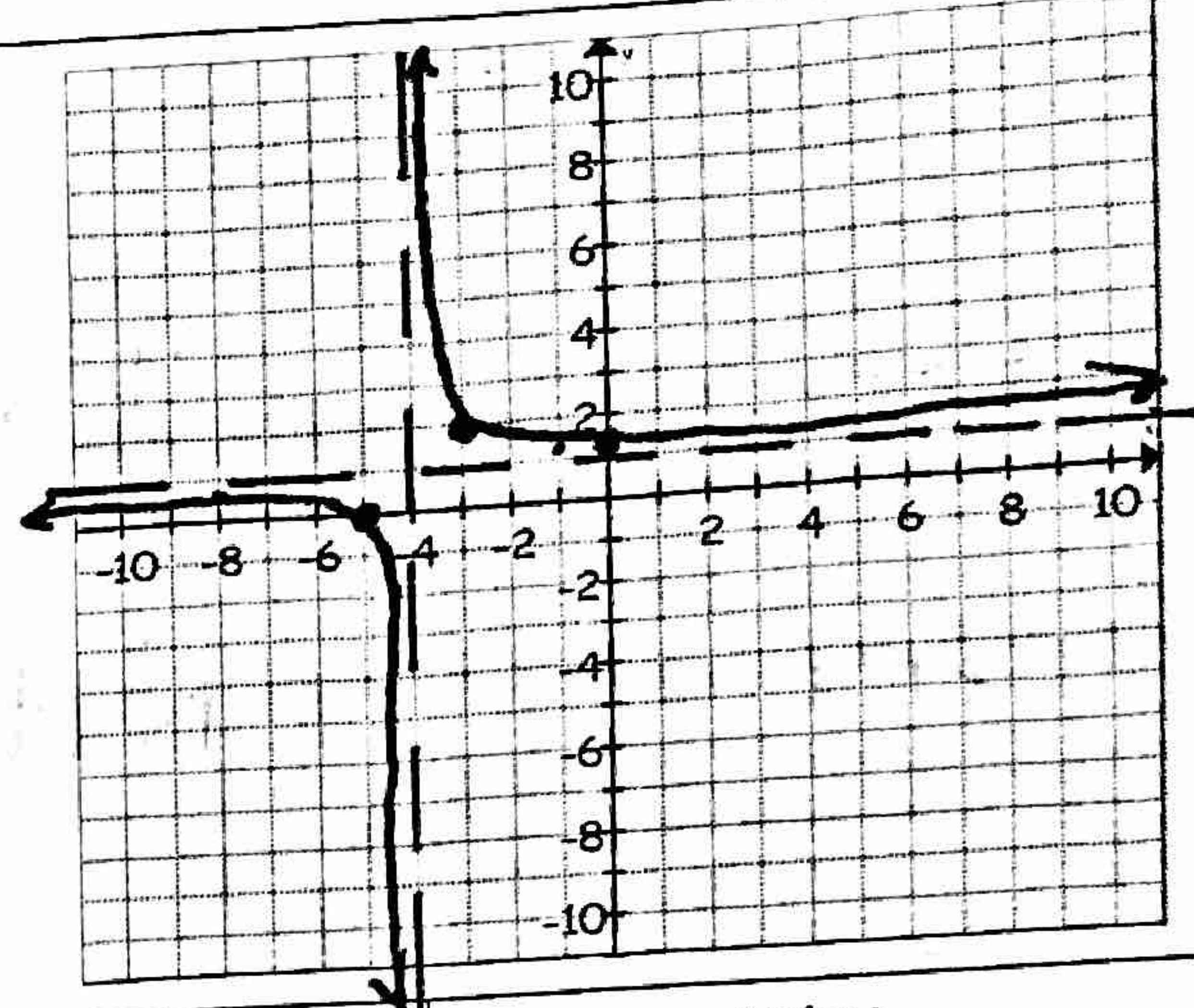
Zero(s)/x-intercept(s):  
 $(-2, 0)$

y-intercept:  
 $(0, -1)$   
 $\frac{4}{-2} + 1$

domain:  
 $(-\infty, 2) \cup (2, \infty)$

range:  
 $(-\infty, 1) \cup (1, \infty)$

11.  
 $y = \frac{x+5}{x+4}$



V asymptote(s):  
 $x=-4$

H asymptote:  
 $y=1$

SLANT asymptote:  
 none

Hole(s):  
 none

Zero(s)/x-intercept(s):  
 $(-5, 0)$

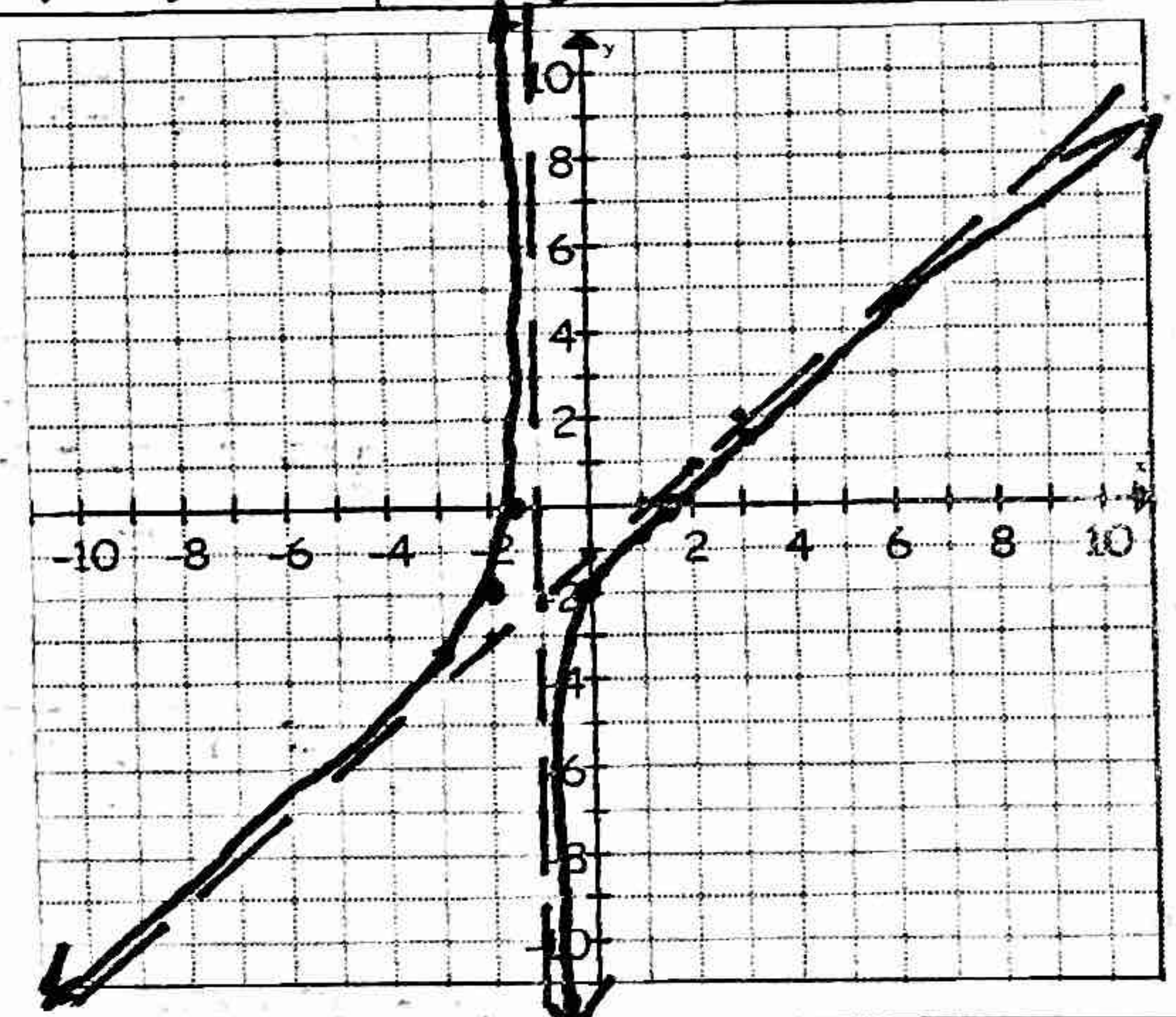
y-intercept:  
 $(0, \frac{5}{4})$

domain:  
 $(-\infty, -4) \cup (-4, \infty)$

range:  
 $(-\infty, 1) \cup (1, \infty)$

12.  
 $y = \frac{x^2 - 2}{x+1}$

$x+1 \overline{) x^2 + x - 2}$   
 $\underline{x^2 + x}$   
 $-x - 2$   
 $\underline{-x - 1}$   
 $-1$



V asymptote(s):  
 $x=-1$

H asymptote:  
 none

SLANT asymptote:  
 $y=x-1$

Hole(s):  
 none

Zero(s)/x-intercept(s):  
 $(\pm 1.4, 0)$

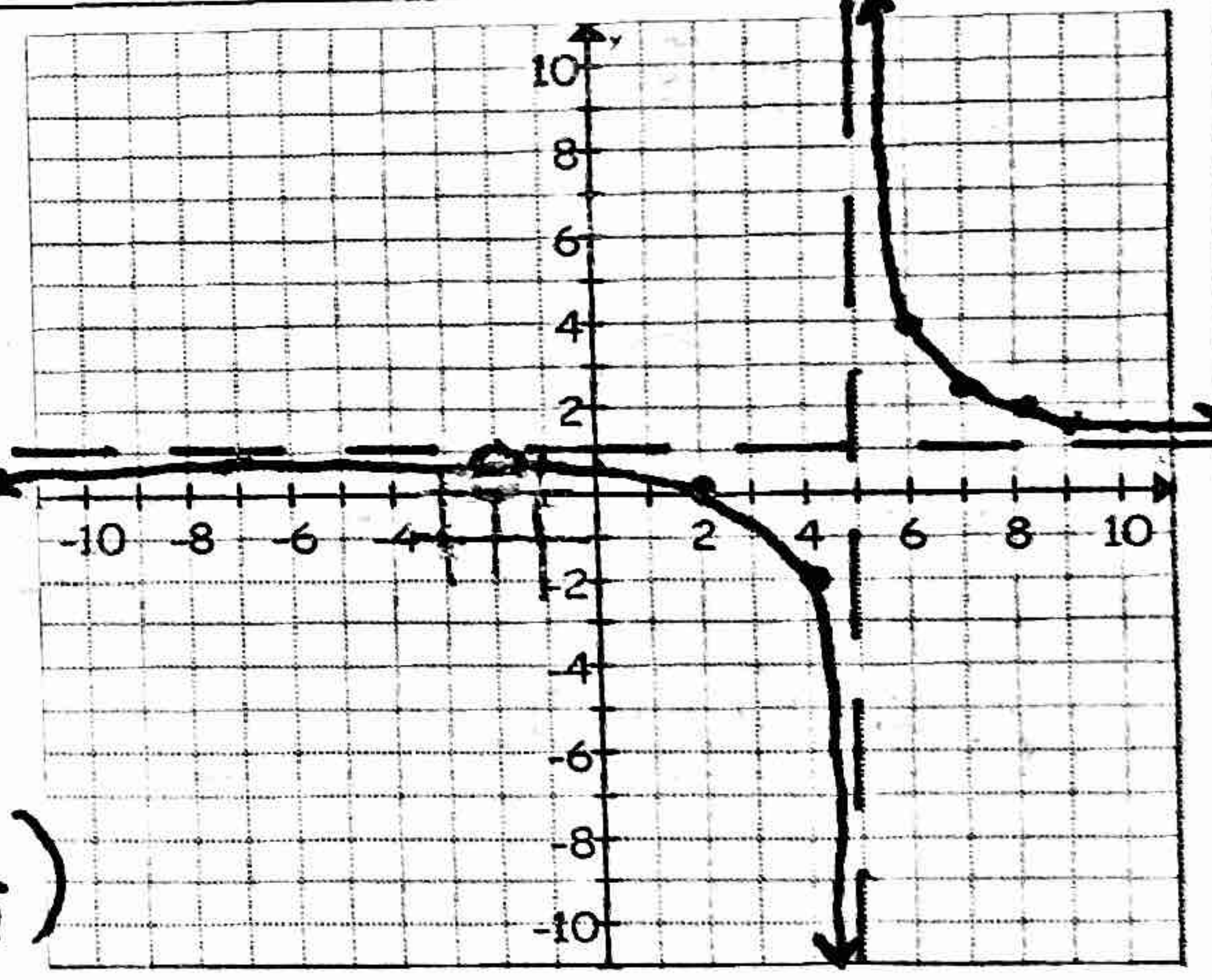
y-intercept:  
 $(0, -2)$

domain:  
 $(-\infty, -1) \cup (-1, \infty)$

range:  
 $(-\infty, \infty)$

13.  
 $y = \frac{x^2 - 4}{x^2 - 3x - 10}$

$\frac{(x+2)(x-2)}{(x-5)(x+2)}$   
 $\frac{-2-2}{-2-5} = \frac{-4}{-7}$   
 hole:  $(-2, \frac{4}{7})$



V asymptote(s):  
 $x=5$

H asymptote:  
 $y=1$

SLANT asymptote:  
 none

Hole(s):  
 $(-2, \frac{4}{7})$

Zero(s)/x-intercept(s):  
 $(2, 0)$

y-intercept:  
 $(0, \frac{4}{10}) = (0, \frac{2}{5})$

domain:  
 $(-\infty, -2) \cup (-2, 5) \cup (5, \infty)$

range:  
 $(-\infty, \frac{4}{7}) \cup (\frac{4}{7}, 1) \cup (1, \infty)$



In #14-16, match the function with its graph.

14.  $y = \frac{-3x^2 + 6x}{x^2 - 6x + 8}$

B

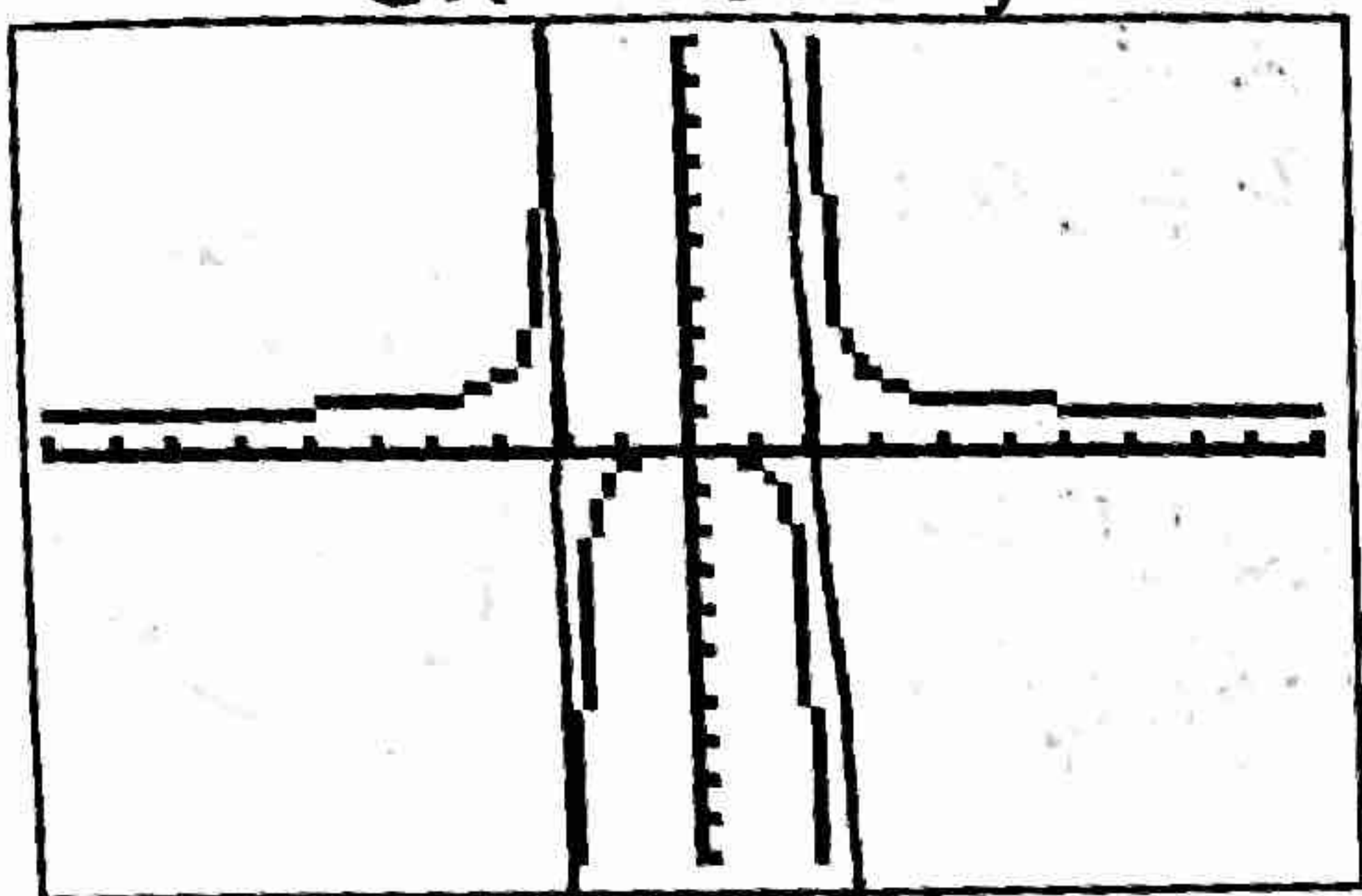
15.  $y = 1 - \frac{x}{x+2}$

C

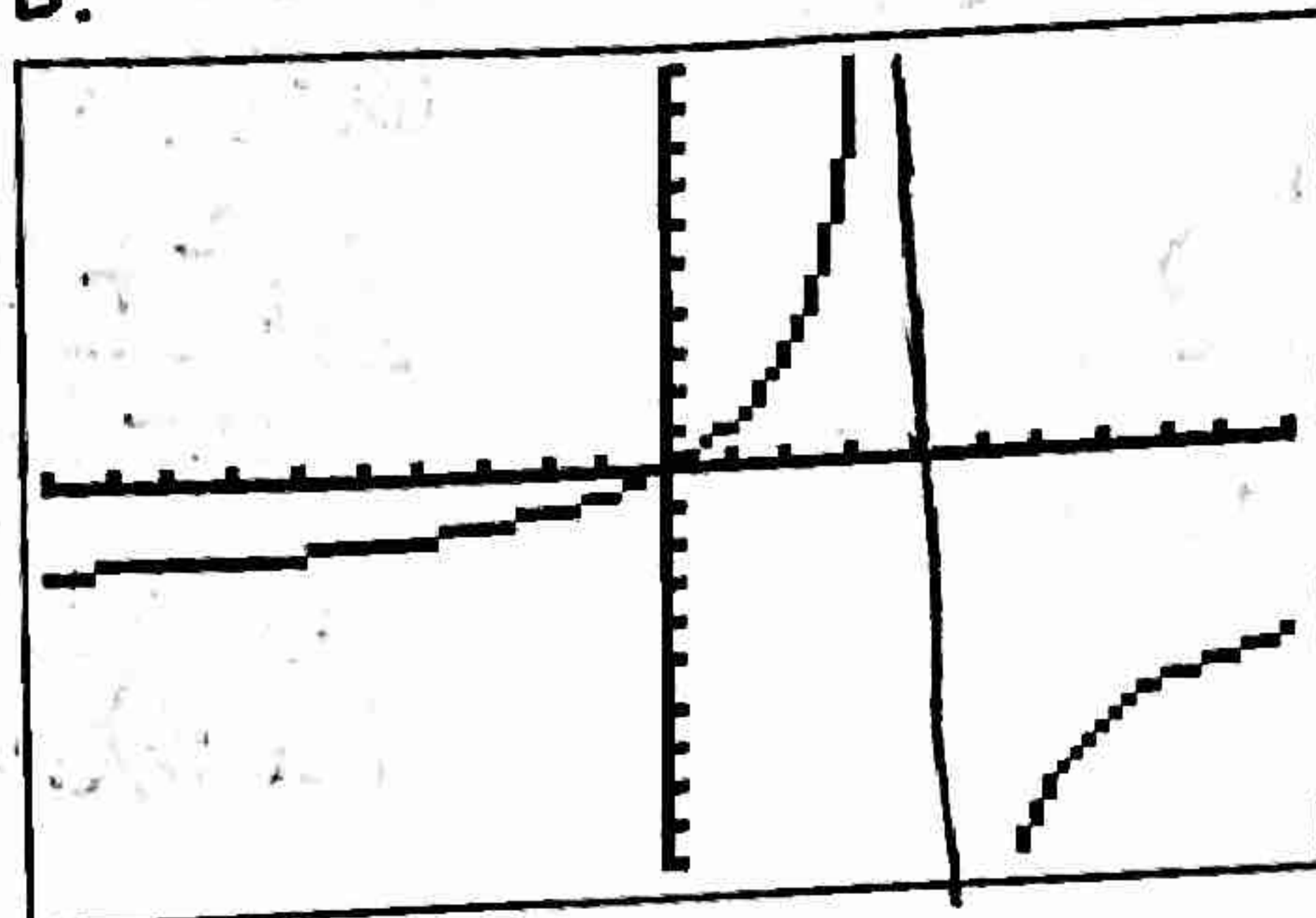
16.  $y = \frac{x^2}{x^2 - 4}$

A

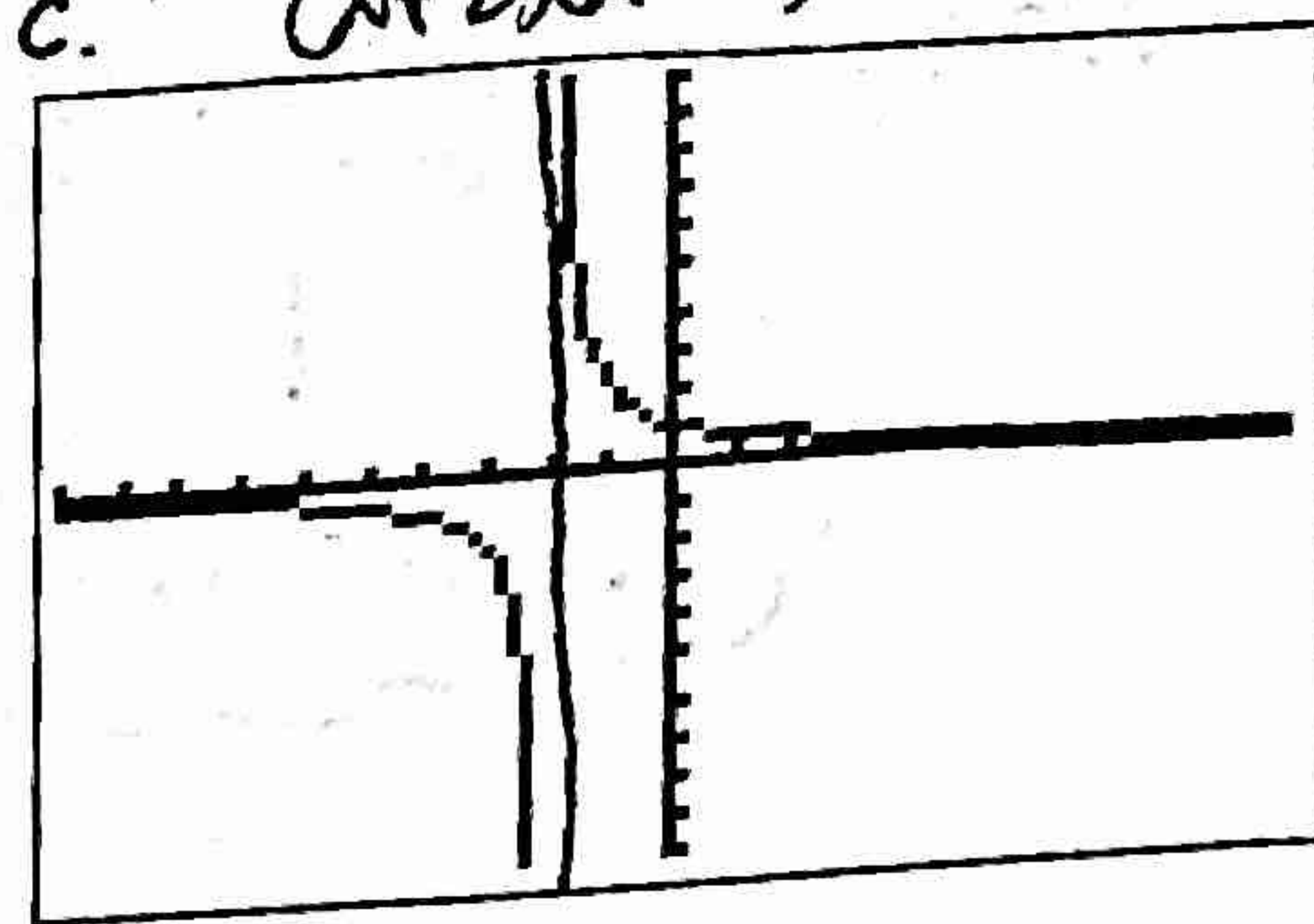
A.  $(x-4)(x-2)$



B.



C.



Given the function  $f(x) = \frac{5}{x}$ , write the equation that would represent the translation that:

17. has a horizontal asymptote  $y = 5$  and a vertical asymptote  $x = 3$ .

$f(x) = \frac{5}{x-3} + 5$

18. has a horizontal asymptote  $y = -8$  and a vertical asymptote  $x = 0$

$f(x) = \frac{5}{x} - 8$

19. has a horizontal asymptote  $y = 0$  and a vertical asymptote  $x = -6$

$f(x) = \frac{5}{x+6}$

20. shifts the function up 2 and left 7.

$f(x) = \frac{5}{x+7} + 2$

Perform the indicated operation and simplify.

21.  $\frac{a^2 - b^2}{a^4(a+2)} \cdot \frac{a^2}{a+b}$

$\frac{(a+b)(a-b) \cdot a \cdot a}{a \cdot a \cdot a \cdot a (a+2)(a+b)}$

$\frac{a-b}{a^2(a+2)}$

22.  $\frac{x^2 - 4}{2x^2 + 7x + 3} \cdot \frac{x^2 + 5x - 14}{2x^2 - 9x - 5}$

$\frac{(x+2)(x-2)}{(2x+1)(x+3)} \cdot \frac{(2x-1)(x-5)}{(x+7)(x-2)}$

$\frac{(x+2)(x-5)}{(x+3)(x+7)}$

23.  $\frac{2x-1}{x^2-x-2} - \frac{1}{x-2}$

$\frac{2x-1}{(x-2)(x+1)} - \frac{(x+1)}{(x-2)(x+1)}$

$\frac{2x-1-x-1}{(x-2)(x+1)} = \frac{(-x-2)}{(x-2)(x+1)}$

$\frac{1}{x+1}$

24.  $\frac{x+7}{2-x} - \frac{x-5}{x-2}$

$\frac{x+7}{2-x} + \frac{x-5}{2-x}$

$\frac{2x+2}{2-x}$  or  $\frac{-2x-2}{x-2}$

$\frac{2(x+1)}{2-x} = \frac{-2(x+1)}{x-2}$



25.  $\frac{1}{x^3 + 10x^2} \div \frac{x^2 - 9}{x + 3} \cdot \frac{x + 10}{x^2 + 7x + 12}$

$$\frac{1}{x^2(x+10)} \cdot \frac{(x+3)}{(x+3)(x-3)} \cdot \frac{(x+10)}{(x+3)(x+4)}$$

$$\frac{1}{x^2(x-3)(x+3)(x+4)}$$

26.  $\frac{2a}{a-3} - \frac{2a}{a+3} + \frac{36}{a^2-9}$

$$\frac{2a(a+3) - 2a(a-3) + 36}{(a-3)(a+3)}$$

$$\frac{2a^2 + 6a - 2a^2 + 6a + 36}{(a-3)(a+3)}$$

$$\frac{12a + 36}{(a-3)(a+3)} = \frac{12(a+3)}{(a-3)(a+3)} = \frac{12}{a-3}$$

27.  $\frac{\frac{3x}{x-3} + \frac{6}{x+2}}{x^2 - x - 6}$

$$\frac{3x(x+2) + 6(x-3)}{(x-3)(x+2)} \cdot \frac{(x-3)(x+2)}{3}$$

$$\frac{3x^2 + 6x + 6x - 18}{(x-3)(x+2)} \cdot \frac{(x-3)(x+2)}{3}$$

$$\frac{3x^2 + 12x - 18}{3} = x^2 + 4x - 6$$

28.  $\frac{\frac{x^2 - 9x - 22}{x^2 + 5x - 24}}{\frac{x+2}{x-3}}$

$$\frac{(x-11)(x+2)}{(x+8)(x-3)} \cdot \frac{(x-3)}{(x+2)} = \frac{x-11}{x+8}$$

Solve each equation. Round any decimals to the nearest hundredth and check your answers.

29.  $2x \left( \frac{2}{x} + \frac{x}{2} \right) = 8$

mult both sides by  $2x$ !

$$4 + x^2 = 16x$$

$$x^2 - 16x + 4 = 0$$

$$\frac{16 \pm \sqrt{256 - 4(4)}}{2} = \frac{16 \pm \sqrt{240}}{2} = \frac{16 \pm 4\sqrt{15}}{2}$$

$$8 \pm 2\sqrt{15}$$

30.  $\frac{x}{x^2 - 8} = \frac{2}{x}$

Cross multiplied

$$x^2 = 2x^2 - 16$$

$$-x^2 = -16$$

$$x^2 = 16$$

$$x = \pm 4$$

31.  $\frac{2x-4}{x-2} = \frac{4}{x-4}$

$$(2x-4)(x-4) = 4(x-2)$$

$$2x^2 - 12x + 16 = 4x - 8$$

$$2x^2 - 16x + 24 = 0$$

$$2(x^2 - 8x + 12) = 0$$

$$2(x-6)(x-2) = 0$$

$x = 6, 2$   
 $x \neq 2$   
so  $x = 6$

32.  $\frac{1}{x-2} + \frac{1}{x+3} = \frac{5}{x^2+x-6}$

$$\frac{x+3 + x-2}{(x-2)(x+3)} = \frac{5}{(x-2)(x+3)}$$

$$2x+1 = 5$$

$$2x = 4$$

$$x = 2 \dots x \neq 2 \dots \text{so } \emptyset$$

If the denom. are the same, then the num. are the same!

33.  $\frac{5x}{x-1} - 2 = \frac{14}{x^2-1}$

$$\frac{5x - 2(x-1)}{x-1} = \frac{14}{(x+1)(x-1)}$$

$$\frac{5x - 2x + 2}{x-1} = \frac{14}{(x+1)(x-1)}$$

$$3x + 2 = \frac{14}{x+1}$$

$$3x^2 + 5x + 2 = 14$$

$$3x^2 + 5x - 12 = 0$$

$$(3x-4)(x+3) = 0$$

$x = \frac{4}{3}, -3$

34.  $\frac{r}{r+4} + \frac{r}{r-4} = \frac{r^2+16}{r^2-16}$

$$\frac{r(r-4) + r(r+4)}{(r+4)(r-4)} = \frac{r^2+16}{(r+4)(r-4)}$$

$$r^2 - 4r + r^2 + 4r = r^2 + 16$$

$$2r^2 = r^2 + 16$$

$$r^2 = 16$$

$$r = \pm 4 \dots \emptyset$$