A2T Unit 5 Test Review
Inverses, Radical Functions, & Relations

Name Master C
Date _____ Block 6

Target 1: I can apply operations with functions, evaluate compositions of functions, and apply composition of functions to real world applications.

1-9: Given the functions below, perform each operation given. *Show all work and circle your final answer.*

$f(x) = 2x - 8$ $f^{-1}(x) = \frac{1}{2}x + 4$ $g(x) = x^2 - 4$ $h(x) = 2|x| + 1$ $m(x) = 2x^2 - 3x + 5$

1. $f(h(-2))$ $h(-2) = 2|-2| + 1$, $2(2)+1 = 5$
$f(5) = 2(5) - 8$
$10 - 8 = \boxed{2}$

2. $f(g(-2x))$ $g(-2x) = (-2x)^2 - 4$, $4x^2 - 4$
$f(4x^2-4) = 2(4x^2-4) - 8$
$8x^2 - 8 - 8$
$\boxed{8x^2 - 16}$

3. $f(g(x+1))$ $g(x+1) = (x+1)^2 - 4$, $x^2+2x+1-4$, x^2+2x-3
$f(x^2+2x-3)$
$2(x^2+2x-3) - 8$
$2x^2+4x-6-8$
$\boxed{2x^2+4x-14}$

4. $f^{-1}(f(4))$ $f(4) = 2(4) - 8 = 8 - 8 = 0$
$f^{-1}(0) = \frac{1}{2}(0) + 4 = 0 + 4 =$
$\boxed{4}$

5. $f(g(x))$
$f(x^2-4)$
$2(x^2-4) - 8$
$2x^2 - 8 - 8 = \boxed{2x^2 - 16}$

6. $(\frac{f}{g})(x)$ $\frac{2x-8}{x^2-4} = \frac{2(x-4)}{(x+2)(x-2)}$
FA: $\boxed{\frac{2x-8}{x^2-4},\ x \neq \pm 2}$

7. $(g + m)(x)$
$x^2 - 4 + 2x^2 - 3x + 5$
$\boxed{3x^2 - 3x + 1}$

8. $(g - m)(x)$
$x^2 - 4 - (2x^2 - 3x + 5)$
$x^2 - 4 - 2x^2 + 3x - 5$
$\boxed{-x^2 + 3x - 9}$

9. $(f \cdot g)(x)$
$(2x-8)(x^2-4)$
$2x^3 - 8x - 8x^2 + 32$
$\boxed{2x^3 - 8x^2 - 8x + 32}$

10: You are going shopping at DSW and see a pair of shoes you like that are 20% off. But you also see that there will be an additional 30% taken off of those shoes. Complete a – e below.

a. Write a function f(x) to calculate the price of your shoes with 20% off.
$f(x) = x - .20x \longrightarrow f(x) = .80x$

b. Write a function g(x) to calculate the price of your shoes with 30% off.
$g(x) = x - .30x \longrightarrow g(x) = .70x$

c. Use the composition of functions to write a function that represents taking 20% off then 30% off.
$g(f(x)) = g(.80x) = .70(.80)x = .56x$

d. Use your function in c to find out the cost of a $90 pair of shoes.
$.56(90) = \$50.40$

e. What would the sale price be if the clerk took 30% off first?
$f(g(x)) = f(.70x) = .80(.70x) = .56x = \$50.40$

Target 2: I can graph a function, including square root functions, and its inverse and identify the domain and range of each; I can find the equation of the inverse of a function; I can verify inverses using composition of functions

11-13: Write the equation of the inverse of each function. *Show your work and circle your answer (use proper notation)!*

11. $f(x) = 3x - 7$
$x = 3y - 7$
$x + 7 = 3y$
$\frac{1}{3}x + \frac{7}{3} = y$
$\boxed{f^{-1}(x) = \frac{1}{3}x + \frac{7}{3}}$

12. $g(x) = x^2 + 3$
$x = y^2 + 3$
$x - 3 = y^2$
$\pm\sqrt{x-3} = y$
$\boxed{y^{-1} = \pm\sqrt{x-3}}$
not a function!

13. $h(x) = 2\sqrt{x-3}$
$x = 2\sqrt{y-3}$
$(\frac{1}{2}x = \sqrt{y-3})^2$
$\frac{1}{4}x^2 = y - 3$
$\boxed{h^{-1}(x) = \frac{1}{4}x^2 + 3 \quad x \geq 0}$
You have to restrict the domain since h(x) only has the $+\sqrt{}$!

14-15: Use the *composition of functions* to determine whether functions f(x) and g(x) are inverses of each other.

14. $f(x) = 6x - 2 \quad g(x) = \frac{1}{6}x + 3$

$f(g(x)) = 6(\frac{1}{6}x+3)-2$
$x + 18 - 2 = x + 16$

$g(f(x)) = \frac{1}{6}(6x-2)+3$
$x - \frac{1}{3} + 3 = x + \frac{8}{3}$
$x - \frac{1}{3} + \frac{9}{3}$

Not inverses b/c $f(g(x)) \neq g(f(x)) \neq x$

15. $f(x) = 2x + 3 \quad g(x) = \frac{1}{2}(x-3) = \frac{1}{2}x - \frac{3}{2}$

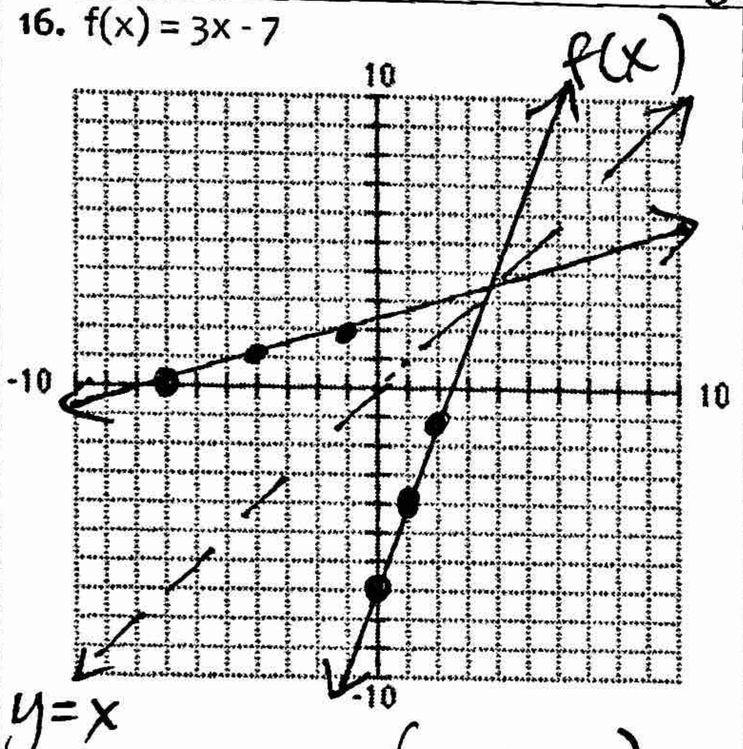
$f(g(x)) = 2(\frac{1}{2}x - \frac{3}{2}) + 3$
$x - 3 + 3 = x$

$g(f(x)) = \frac{1}{2}(2x+3) - \frac{3}{2}$
$x + \frac{3}{2} - \frac{3}{2} = x$

They are inverses b/c $f(g(x)) = g(f(x)) = x$!

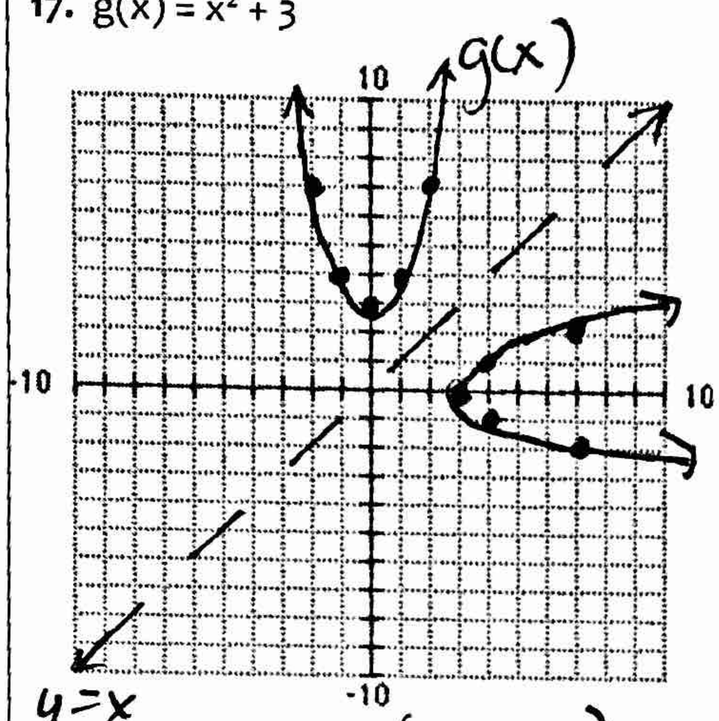
16-18: a. Graph the function in pencil and its inverse in colored pencil, pen or highlighter. Label each graph.
b. Graph and label the line of reflection using dashed line.
c. State the domain and range of both in interval notation, and state if the INVERSE is a function.

16. $f(x) = 3x - 7$



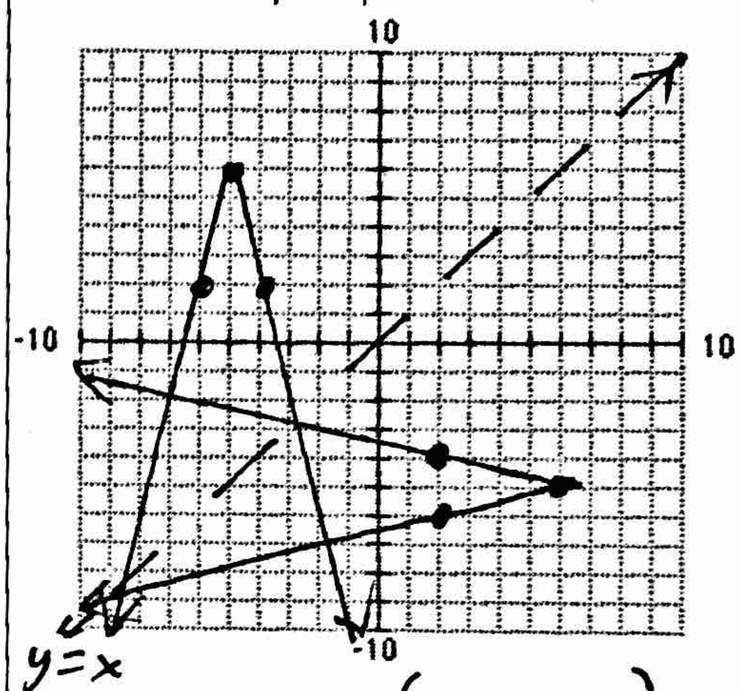
Function: Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Inverse: Domain: $(-\infty, \infty)$
Range: $(-\infty, \infty)$
Is the Inverse a function: (YES) or NO

17. $g(x) = x^2 + 3$



Function: Domain: $(-\infty, \infty)$
Range: $[3, \infty)$
Inverse: Domain: $[3, \infty)$
Range: $(-\infty, \infty)$
Is the Inverse a function: YES or (NO)

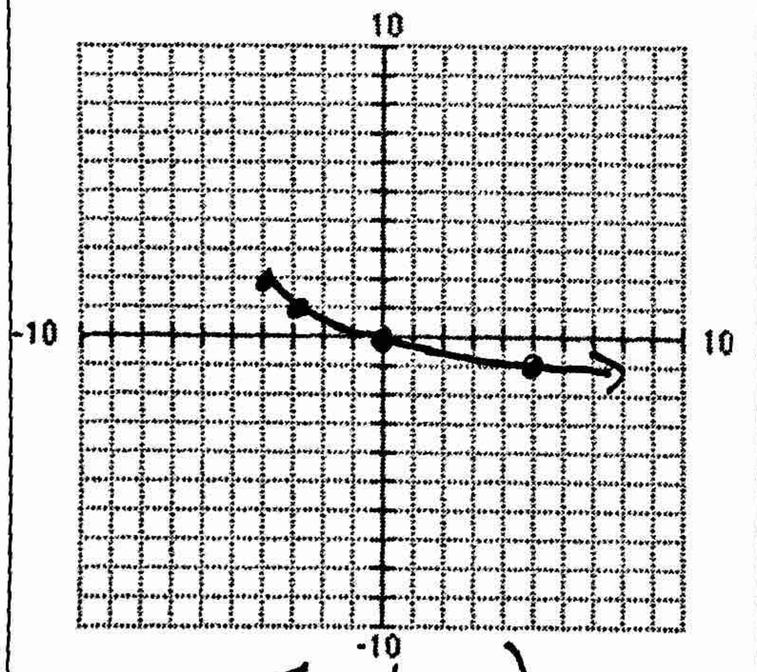
18. $h(x) = -4|x+5| + 6$



Function: Domain: $(-\infty, \infty)$
Range: $(-\infty, 6]$
Inverse: Domain: $(-\infty, 6]$
Range: $(-\infty, \infty)$
Is the Inverse a function: YES or (NO)

19-21: Graph each function *without a calculator* and state the domain and range in interval notation.

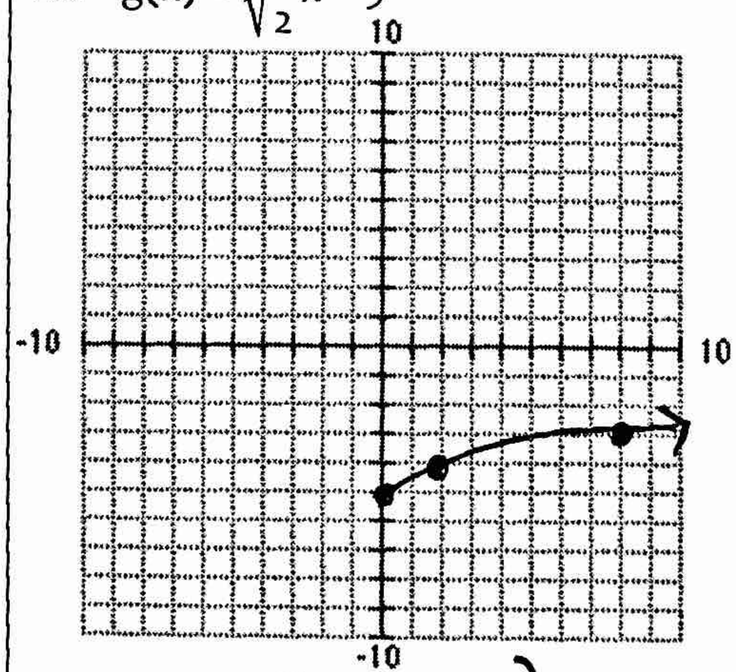
19. $f(x) = -\sqrt{x+4} + 2$



Domain: $[-4, \infty)$
Range: $(-\infty, 2]$

x	y
-4	2
-3	1
0	0
5	-1

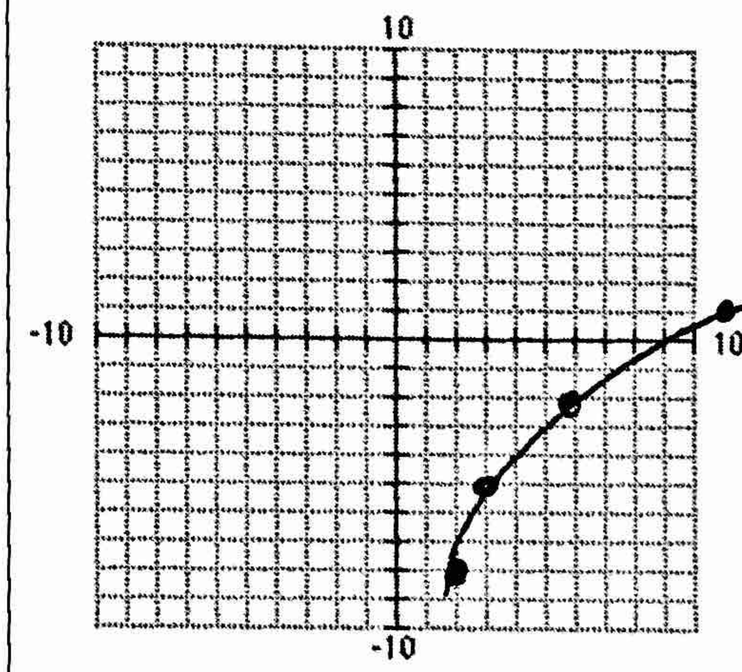
20. $g(x) = \sqrt{\frac{1}{2}x - 5}$



Domain: $[0, \infty)$
Range: $[-5, \infty)$

x	y
0	-5
2	-4
8	-3

21. $r(x) = 3\sqrt{x-2} - 8$



Domain: $[2, \infty)$
Range: $[-8, \infty)$

x	y
2	-8
3	-5
6	-2

Target 3: I can simplify expressions containing rational exponents and radicals of a variety of indices.

22-23: Write each expression in rational exponent form. *Circle your final answer.*

22. $4\sqrt[3]{8a^4b^6c^2}$ $4 \cdot 2a^{\frac{4}{3}}b^{\frac{6}{3}}c^{\frac{2}{3}}$ $8a^{\frac{4}{3}}b^2c^{\frac{2}{3}}$

23. $\sqrt{a^3b^4c^7}$ $a^{\frac{3}{2}}b^{\frac{4}{2}}c^{\frac{7}{2}}$ $a^{\frac{3}{2}}b^2c^{\frac{7}{2}}$

24-25: Write each expression in simplified radical form. *Circle your final answer.*

24. $x^{\frac{1}{2}}y^{\frac{2}{3}}z^{\frac{1}{6}}$ $x^{\frac{3}{6}}y^{\frac{4}{6}}z^{\frac{1}{6}}$ $\sqrt[6]{x^3y^4z}$

25. $9^{\frac{4}{3}}x^{\frac{1}{2}}y^{\frac{5}{6}}$ $9^{\frac{8}{6}}x^{\frac{3}{6}}y^{\frac{5}{6}} = \sqrt[6]{9^8x^3y^5}$ $\sqrt[6]{9^6 \cdot 9^2x^3y^5} = 9\sqrt[6]{81x^3y^5}$

26-40: Simplify each expression. Write your final answer in simplified radical form. *Circle your final answer.*

26. $25^{\frac{1}{2}}$ $\sqrt{25}$ 5

27. $8^{\frac{2}{3}}$ $(\sqrt[3]{8})^2$ $(2)^2$ 4

28. $9^{-\frac{5}{2}}$ $\frac{1}{9^{\frac{5}{2}}}$ $\frac{1}{(\sqrt{9})^5} = \frac{1}{3^5}$ $\frac{1}{243}$

29. $4^{\frac{1}{3}} \cdot 4^{\frac{2}{5}}$ $4^{\frac{1}{3}+\frac{2}{5}}$ $4^{\frac{5}{15}+\frac{6}{15}}$ $4^{\frac{11}{15}}$ $\sqrt[15]{4^{11}} = \sqrt[15]{4194304}$

30. $\frac{\sqrt{20}}{\sqrt{5}}$ $\sqrt{\frac{20}{5}}$ $\sqrt{4}$ 2

31. $\sqrt[5]{96} - 4\sqrt[5]{3}$ $\sqrt[5]{32 \cdot 3} - 4\sqrt[5]{3}$ $2\sqrt[5]{3} - 4\sqrt[5]{3}$ $-2\sqrt[5]{3}$

32. $(6^{\frac{3}{4}})^{\frac{1}{3}}$ $6^{\frac{1}{4}}$ $\sqrt[4]{6}$

33. $\sqrt[4]{\frac{x^{10}}{y^8}}$ $\frac{\sqrt[4]{x^{10}}}{y^2}$ $\frac{\sqrt[4]{x^8 \cdot x^2}}{y^2} = \frac{x^2\sqrt[4]{x^2}}{y^2}$ or $\frac{x^2\sqrt{x}}{y^2}$

34. $a^{\frac{2}{3}} \cdot a^{\frac{1}{4}}$ $a^{\frac{2}{3}+\frac{1}{4}}$ $a^{\frac{8}{12}+\frac{3}{12}}$ $a^{\frac{11}{12}}$ $\sqrt[12]{a^{11}}$

35. $\left(16^{\frac{4}{3}}\right)^{-\frac{3}{8}} = 16^{-\frac{4}{8}}$ $16^{-\frac{1}{2}} = \frac{1}{\sqrt{16}}$ $\frac{1}{4}$

36. $\sqrt[6]{\frac{a^2}{2}} \cdot \frac{\sqrt[6]{25}}{\sqrt[6]{25}}$ $= \frac{\sqrt[6]{32a^2}}{2}$

37. $64^{\frac{1}{4}}$ $\sqrt[4]{64}$ $\sqrt[4]{16 \cdot 4}$ $2\sqrt[4]{4}$ or $2\sqrt{2}$

38. $(2\sqrt[3]{54})(3\sqrt[3]{18})$ $6\sqrt[3]{54 \cdot 18}$ $6\sqrt[3]{27 \cdot 2 \cdot 2 \cdot 9}$ $6 \cdot 3\sqrt[3]{36}$ $18\sqrt[3]{36} = \sqrt[3]{6^2}$

39. $\sqrt[3]{-640a^5b^2c^9}$ $\sqrt[3]{-64 \cdot 10a^3a^2b^2c^9}$ $-4ac^3\sqrt[3]{10a^2b^2}$

40. $3\sqrt{2} + \sqrt{8} - 2\sqrt{32} + \sqrt{12}$ $3\sqrt{2} + \sqrt{4 \cdot 2} - 2\sqrt{16 \cdot 2} + \sqrt{4 \cdot 3}$ $3\sqrt{2} + 2\sqrt{2} - 8\sqrt{2} + 2\sqrt{3}$ $-3\sqrt{2} + 2\sqrt{3}$

Target 4: I can solve equations containing rational exponents or radicals.

41-50: Solve each radical equation. Check for extraneous solutions. You may use your calculator! *Circle your final answer.*

41. $5x^{\frac{3}{2}} = 40$

$(x^{\frac{3}{2}})^{\frac{2}{3}} = (8)^{\frac{2}{3}}$

$x = (\sqrt[3]{8})^2$

$x = 2^2$

$x = 4$

✓: $5(4)^{\frac{3}{2}} = 40$

$5(\sqrt{4})^3 = 40$

$5(2)^3 = 40$

$5(8) = 40$

$40 = 40$ ✓

42. $\sqrt[3]{x} + 5 = 9$

$(\sqrt[3]{x} = 4)^3$

$x = 4^3$

$x = 64$

✓: $\sqrt[3]{64} + 5 = 9$

$4 + 5 = 9$

$9 = 9$ ✓

43. $2\sqrt[3]{1-3x} + 4 = 6$

$2\sqrt[3]{1-3x} = 2$

$(\sqrt[3]{1-3x} = 1)^3$

$1 - 3x = 1$

$-3x = 0$

$x = 0$

✓: $2\sqrt[3]{1-3(0)} + 4 = 6$

$2\sqrt[3]{1} + 4 = 6$

$2 + 4 = 6$

$6 = 6$ ✓

44. $7x\sqrt{3} - 5 = 0$

$7x\sqrt{3} = 5$

$\frac{x\sqrt{3}}{\sqrt{3}} = \frac{5}{7\sqrt{3}}$

$x = \frac{5}{7\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{21}$

$\approx .41$

✓: $7\left(\frac{5\sqrt{3}}{21}\right)\sqrt{3} - 5 = 0$

$\frac{5\sqrt{3}}{3}\sqrt{3} - 5 = 0$

$\frac{5 \cdot 3}{3} - 5 = 0$

$5 - 5 = 0$ ✓

45. $\sqrt[4]{7x-2} + 12 = 7$

$\sqrt[4]{7x-2} = -5$

$\emptyset$

even root can't = neg #

or

$7x - 2 = 5^4$

$7x - 2 = 625$

$7x = 627$

$x = \frac{627}{7}$

✓: $\sqrt[4]{7\left(\frac{627}{7}\right) - 2} + 12 = 7$

$\sqrt[4]{627} - 2 + 12 = 7$

$\sqrt[4]{625} + 12 \neq 7$

46. $\sqrt[3]{4x-1} = 3$

$(\sqrt[3]{4x-1})^3 = (3)^3$

$4x - 1 = 27$

$4x = 28$

$x = 7$

✓: $\sqrt[3]{4(7)-1} = 3$

$\sqrt[3]{28-1} = 3$

$\sqrt[3]{27} = 3$ ✓

47. $(\sqrt{3x+15} = \sqrt{x+5})^2$

$3x + 15 = x + 5$

$2x = -10$

$x = -5$

✓: $\sqrt{3(-5)+15} = \sqrt{-5+5}$

$\sqrt{0} = \sqrt{0}$

$0 = 0$ ✓

48. $(\sqrt{3x+13} = x + 5)^2$

$3x + 13 = x^2 + 10x + 25$

$x^2 + 7x + 12 = 0$

$(x+4)(x+3) = 0$

$x = -4, -3$

✓: $\sqrt{3(-4)+13} = -4 + 5$

$\sqrt{-12+13} = 1$

$\sqrt{1} = 1$ ✓

$\sqrt{3(-3)+13} = -3 + 5$

$\sqrt{-9+13} = 2$

$\sqrt{4} = 2$

$2 = 2$ ✓

49. $(\sqrt{6x-4} = \sqrt{2x+10})^2$

$6x - 4 = 2x + 10$

$4x = 14$

$x = \frac{7}{2}$ or 3.5

✓: $\sqrt{6\left(\frac{7}{2}\right)-4} = \sqrt{2\left(\frac{7}{2}\right)+10}$

$\sqrt{21-4} = \sqrt{7+10}$

$\sqrt{17} = \sqrt{17}$ ✓

50. $\sqrt{4x^2-3x+2} - 2x - 5 = 0$

$(\sqrt{4x^2-3x+2} = 2x + 5)^2$

$4x^2 - 3x + 2 = 4x^2 + 20x + 25$

$-23 = 23x$

$-1 = x$

✓: $\sqrt{4(-1)^2-3(-1)+2} - 2(-1) - 5 = 0$

$\sqrt{4+3+2} + 2 - 5 = 0$

$\sqrt{9} + 2 - 5 = 0$

$3 + 2 - 5 = 0$

$0 = 0$ ✓