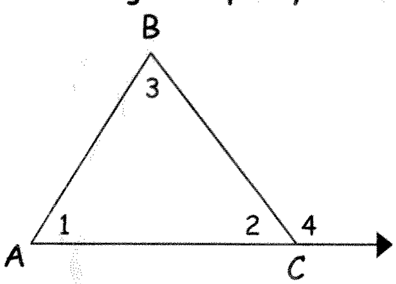
5.3 Inequalities in One Triangle

Name Master
Date ______ Block 4

Inequalities of a Triangle	Definition	Illustration
KEY CONCEPT:	For any real numbers a and b, a > b if and only if there is a positive number c such that a = b + c.	If 6 = 4 + 2, then 6 > 4 and 6 > 2.

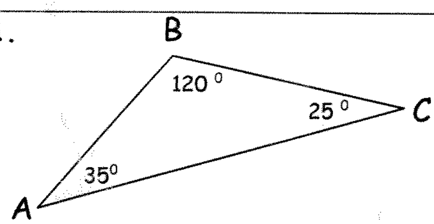
PROPERTIES OF INEQUALITIES FOR REAL NUMBERS: For all numbers a, b, and c
YOU MUST MEMORIZE THESE PROPERTIES AND BE ABLE TO APPLY THEM!

Comparison	a < b, a = b, or a > b.
Transitive	▪ If a < b and b < c, then a < c. ▪ If a > b and b > c, then a > c.
Addition & Subtraction	▪ If a > b, then a + c > b + c and a - c > b - c. ▪ If a < b, then a + c < b + c and a - c < b - c.
Exterior Angle Inequality Theorem 	▪ The measure of an exterior angle of a triangle is greater than the measure of either of its corresponding remote interior angles. ▪ Since m∠4 = m∠1 + m∠3, then it would make sense that m∠4 > m∠1 and m∠4 > m∠3

Angle-Side Relationships in Triangles

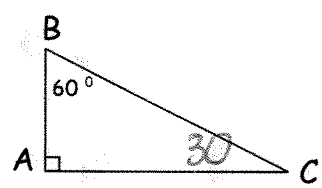
- If one side of a triangle is longer than another side, then the angle opposite the longer side has a greater measure than the angle opposite the other side.
- If one angle of a triangle has a greater measure than another angle, then the side opposite the greater angle is larger than the side opposite the lesser angle.

Ordering triangle angle measures and triangle side lengths

1.


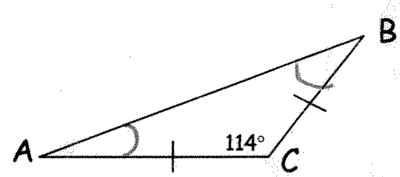
m∠C < m∠A < m∠B

AB < BC < AC

2.


m∠C < m∠B < m∠A

AB < AC < BC

3.


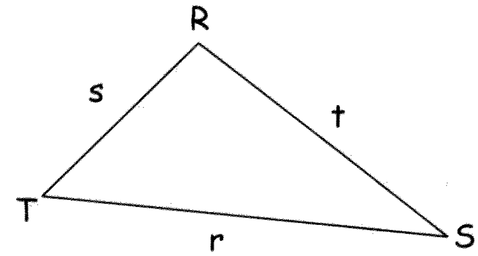
m∠A = m∠B < m∠C

BC = AC < AB

5-5 The Triangle Inequality Theorem Key

While a triangle is formed by three segments, a special relationship must exist among the lengths of the segments in order for them to form a triangle.

1. **Triangle Inequality Theorem:**



> **The sum of the lengths of two sides of a triangle must be greater than the third side.**
>
> **To test for a Triangle: The sum of the 2 smallest sides must be greater than the 3^{rd} side.**

$r+s>t$
$s+t>r$
$r+t>s$

Are these the sides of a triangle?

A. 2, 3, and 5? $2+3>5$? No

B. 2, 2, and 2? $4>2$ yes

C. 3, 3.5, and 8? $6.5 \not> 8$ ~~yes~~ No

D. 4 , $4\sqrt{2}$, 8 $4+4\sqrt{2}>8$? yes

2. **Use the Triangle Inequality Theorem to find a possible side length of a Triangle:**
To find the possible side length of the 3^{rd} side of a triangle when given 2 of the 3 sides of the triangle, you write the following inequality, where x is the 3^{rd} side:

> (the difference of the given 2 sides) < x < (the sum of the given 2 sides)
> (side 1 - side 2) (side 1 + side 2)

Example: Describe the possible lengths of the 3^{rd} side of a triangle if one side is 10 inches and the other side is 7 inches.

$$(10-7) < x < (10+7)$$
$$3 < x < 17$$

A. Could 3 be a possible side of the triangle? No $x>3$!

B. Could 13 be a possible side of the triangle? yes

C. Could 23 be a possible side of the triangle? No $x<17$!

Two sides of a triangle are given. Describe the possible lengths of the third side.

3. 2 cm and 5 cm $3<x<7$

4. 7 in and 12 in $5<x<19$

5. 4 ft and 10 ft $6<x<14$

6. 11 m and 10 m $1<x<21$