5-5 Completing the Square

Master

Finding the Maximum or minimum value of a quadratic function is often needed to answer questions about the data. If the equation is given in vertex form, it is easy to see the vertex. But when the equation is in general/standard form, it needs to be converted to vertex form by completing the square.

WHAT IS A SQUARE?	EXAMPLES: Factor each:	EXAMPLES: Create a perfect square:
When you square a binomial, you get a ______!	1. $x^2 + 10x + 25 = (x+5)^2$	4. $x^2 + 8x + 16 = (x + 4)^2$
$(a + b)^2 = a^2 + 2ab + b^2$	2. $y^2 - 20y + 100 = (y-10)^2$	5. $x^2 - 3x + 2.25 = (x - 1.5)^2$
$(a - b)^2 = a^2 - 2ab + b^2$	3. $4x^2 + 28x + 49 = (2x+7)^2$	6. $16x^2 - 72x + 81 = (4x - 9)^2$

In order to solve a quadratic, you need to be able to manipulate different forms of an equation to get it into the best possible form for solving. Taking the square root of both sides is an easy method, but you can only do this when you have a perfect square trinomial on one side or when the quadratic is in vertex form.

How to solve with the Square Root Property:

A. Factor the perfect square trinomial into a binomial squared.
B. Take the square root of both sides.
C. Make sure you write ± in front of the radical.
D. Simplify the radical.
E. Get x alone by adding or subtracting the number to the other side.
F. Write two equations and solve.
G. Roots that are irrational may be written as exact answers in simplified radical form or they can be written as approximate answers in decimal form when a calculator is used.

1. $x^2 + 6x + 9 = 36$

$\{-9, 3\}$

$(x+3)^2 = 36$

$\sqrt{(x+3)^2} = \pm\sqrt{36}$

$x+3 = \pm 6$

$x = -3 \pm 6 = -3+6, -3-6$

$\boxed{x = 3, -9}$

2. $x^2 - 10x + 25 = 27$

$(x-5)^2 = 27$

$\sqrt{(x-5)^2} = \pm\sqrt{27}$

$x - 5 = \pm 3\sqrt{3}$

$\boxed{x = 5 \pm 3\sqrt{3}}$

Remember: You can put a quadratic function from general form into vertex form by using the method we learned in 5-1. Once you get it into vertex form, you can solve it using the Square root Property above.

How to solve without completing the square:

A. Find the x-coordinate (h) by using the formula: $x = \frac{-b}{2a}$.
B. Find the y-coordinate (k) by substituting the value for x back into the equation and solve for y.
C. Finally, replace a, h, and k into the vertex form of the equation: <u>$y = a(x-h)^2 + k$.</u>
D. Move the k over to the other side.
E. Divide by a.
F. Solve with the Square Root Property.

3. $y = x^2 + 14x + 50$

$x = -\frac{14}{2} = -7$

$y = (-7)^2 + 14(-7) + 50$

$y = 1$

$y = (x+7)^2 + 1$

$0 = (x+7)^2 + 1$

$\sqrt{-1} = \sqrt{(x+7)^2}$

$x + 7 = \pm i$

$\boxed{x = -7 \pm i}$

4. $y = 2x^2 - 16x + 24$

$x = \frac{16}{4} = 4$

$y = 2(4)^2 - 16(4) + 24$

$y = -8$

$y = 2(x-4)^2 - 8$

$0 = 2(x-4)^2 - 8$

$\frac{8}{2} = \frac{2(x-4)^2}{2}$

$\sqrt{4} = \sqrt{(x-4)^2}$

$x - 4 = \pm 2$

$x = 4 \pm 2 = 4+2 = 6$

$= 4-2 = 2$

$\boxed{x = 2, 6}$

How to Solve by Completing the Square (a = 1):

A. Put the equation into standard form and replace the y with 0. Then move c to the other side of the equation.
B. Create a trinomial square: divide the coefficient of x (b) by 2 and square the result $\left(\frac{b}{2}\right)^2$, which will be your c value.
C. Add this c value to both sides of your equation.
D. Rewrite the trinomial square as a binomial squared and simplify the right side of the equation.
E. Then solve the equation using the Square Root Property.

5. $y = x^2 - 2x - 2$

$0 = x^2 - 2x - 2$

$2 = x^2 - 2x$

$x^2 - 2x + \underline{1} = 2 + \underline{1}$

$\sqrt{(x-1)^2} = \sqrt{3}$

$x - 1 = \pm\sqrt{3}$

$\boxed{x = 1 \pm \sqrt{3}}$

6. $y = x^2 - 14x + 1$

$0 = x^2 - 14x + 1$

$-1 = x^2 - 14x$

$x^2 - 14x + \underline{49} = -1 + \underline{49}$

$\sqrt{(x-7)^2} = \sqrt{48}$

$x - 7 = \pm\sqrt{48}$

$\boxed{x = 7 \pm 4\sqrt{3}}$

How to Solve by Completing the Square (a ≠ 1):

A. Put the equation into standard form and replace the y with 0. Then move c to the other side of the equation.
B. Divide both sides by a.
C. Follow steps B-E above to finish solving for x.

$\frac{\sqrt{12}}{\sqrt{5}} = \frac{\sqrt{4}\cdot\sqrt{3}}{\sqrt{5}} = \frac{2\sqrt{3}}{\sqrt{5}}$

$\frac{2\sqrt{3}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$

$x + 1 = \pm\frac{\sqrt{12}}{\sqrt{5}}$

$\boxed{x = -1 \pm \frac{2\sqrt{15}}{5}}$

7. $y = 2x^2 - 4x - 10$

$0 = 2x^2 - 4x - 10$

$\frac{10}{2} = \frac{2x^2}{2} - \frac{4x}{2}$

$5 = x^2 - 2x$

$x^2 - 2x + \underline{1} = 5 + \underline{1}$

$\sqrt{(x-1)^2} = \sqrt{6}$

$x - 1 = \pm\sqrt{6}$

$\boxed{x = 1 \pm \sqrt{6}}$

8. $y + 10 - 8x = 5x^2 + 2x + 3$

$y = 5x^2 + 10x - 7$

$0 = 5x^2 + 10x - 7$

$\frac{7}{5} = \frac{5x^2}{5} + \frac{10x}{5}$

$\frac{7}{5} = x^2 + 2x$

$x^2 + 2x + \underline{1} = \frac{7}{5} + \frac{5}{5}$

$\sqrt{(x+1)^2} = \sqrt{\frac{12}{5}}$

RECAP:

- How many ways do you know how to solve a quadratic equation? List them below.

1) By looking at the graph/where it crosses the x-axis
2) by factoring
3) by completing the square

- What's the easiest method in your opinion?